

Required Homework For Math 108

Each problem below is designed to make you think. Your best ten will count as 10% of your course grade. Show your work and write neatly.

Assignment 1. (Due Jan 21) Card Trick Problem. You are to explain mathematically why the card trick problem done in class works. Write clearly in complete sentences. Here is the trick:

Step 1. Secretly write down the bottom card.

Step 2. Deal 12 cards off the top. Ask the other person to pick any four.

Step 3. Turn over those four and put the rest on the bottom of the deck.

Step 4. On each of these cards count out ten minus the value of the card, with all face cards counting as ten. (For example, you'd put six cards on a four, and none on a king.)

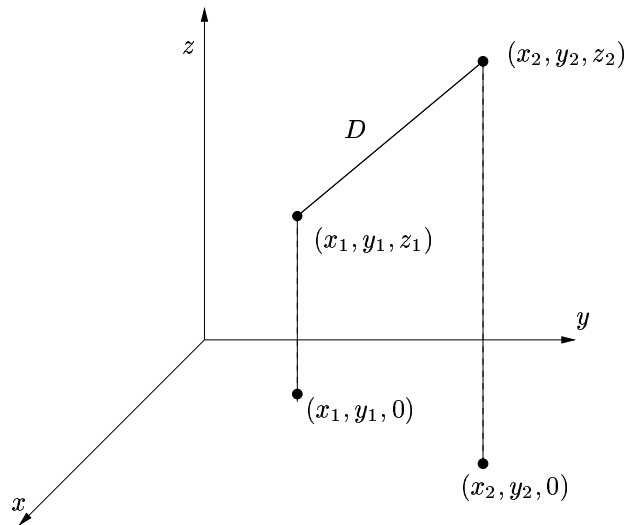
Step 5. Add up the values of the four picked cards. Count out this many cards, face down.

Step 6. Turn over the last card. It will be the card you wrote down in Step 1. Your mission is to find out why.

Hint: There are 52 cards in a deck. Count them.

Assignment 2. (Due Jan 26) Space! Use the Pythagorean Theorem and the distance formula in the plane to prove the distance formula for three dimensions.

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$



Assignment 3. (Due Feb 2) A quadrilateral is a four sided polygon in the plane. Prove that if you connect the midpoints of the four sides of a quadrilateral the resulting figure is parallelogram. Hint: Let (a, b) , (c, d) , (e, f) , and (g, h) , be the four vertices (corners) of an arbitrary quadrilateral. Now, you can compute the midpoints and the necessary slopes.

Assignment 4. (Due Feb 9)

Problem 1: Find a linear function $f(x) = mx + b$ such that $f(1) = 0$ and $f(0) = 1$.

Problem 2: Graph $y = ||2x - 3| - 4|$. Label all intersects. (No, this is not an endorsement of the president's policies.)

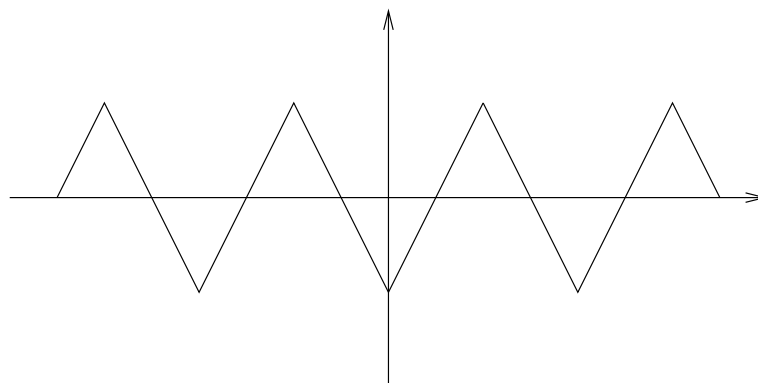
Assignment 5. (Due Feb 16) Let $f(x)$ be a function on the real line.

(a) Show that $g(x) = \frac{f(x) + |f(x)|}{2}$ is a nonnegative function.

(b) Show that $h(x) = \frac{f(x) - |f(x)|}{2}$ is a nonpositive function.

(c) Show that $f(x) = g(x) + h(x)$.

(d) Let $f(x)$ be determined by the graph below. Graph $g(x)$ and $h(x)$ as defined above.



Assignment 6. (Due Feb 23) Even and odd functions.

Problem 1: Suppose that $f(x)$ is an odd function. What is $f(0)$? Prove your claim.

Problem 2: Find a function that is both even and odd.

Problem 3: Let $f(x)$ be a function on the real line.

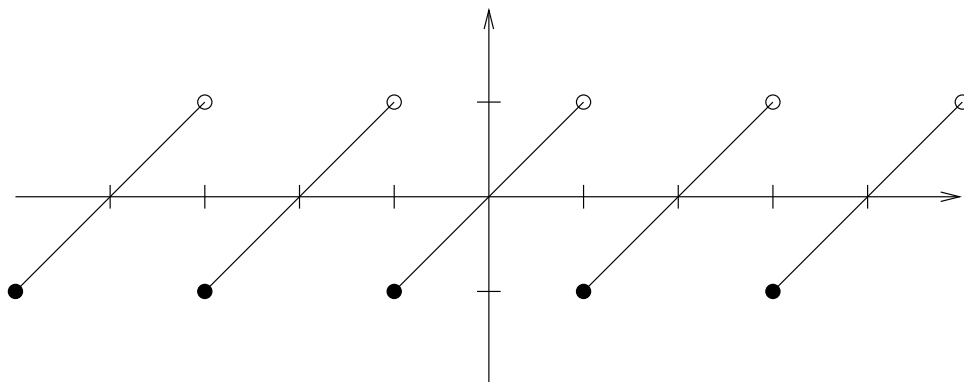
Show that $g(x) = \frac{f(x) + f(-x)}{2}$ is an even function.

Show that $h(x) = \frac{f(x) - f(-x)}{2}$ is an odd function.

Show that $f(x) = g(x) + h(x)$.

Thus, every function can be decomposed into even and odd parts.

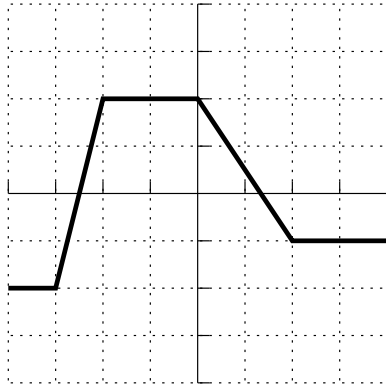
Assignment 7. (Due March 1) Let $f(x) = x^2$ and let $g(x)$ be defined by the graph below, where each tick mark is one unit. Graph $f(g(x))$.



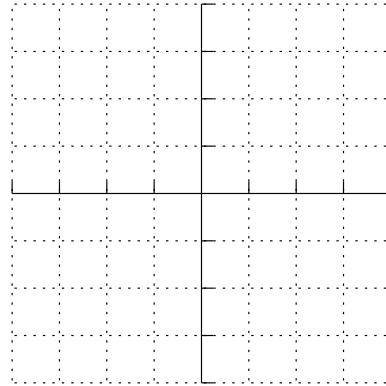
Assignment 8. (Due March 8) Shade in the region of the plane where $(x^2 + y^2 - 9)(x^2 + y^2 - 4) \geq 0$. Hints: Do not multiply it out. Start by asking, what is the graph when we replace the \geq with $=$?

Assignment 9. (Due March 22) Let $f(x)$ be determined by the first graph below; a tick mark is one unit. Graph each of the following.

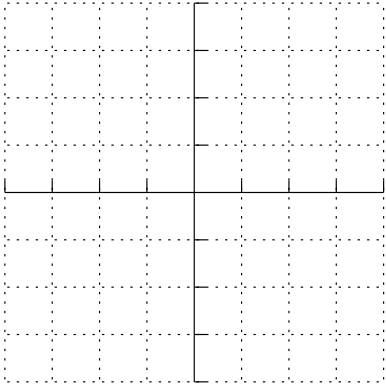
$y = f(x)$ (Given)



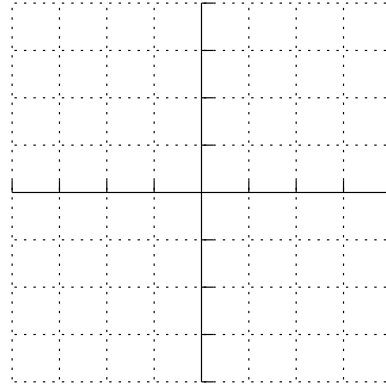
(a) $y = -f(-x)$



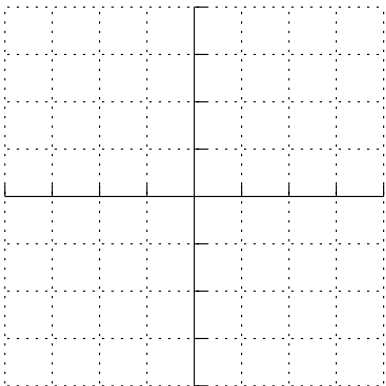
(b) $y = |f(|x|)|$



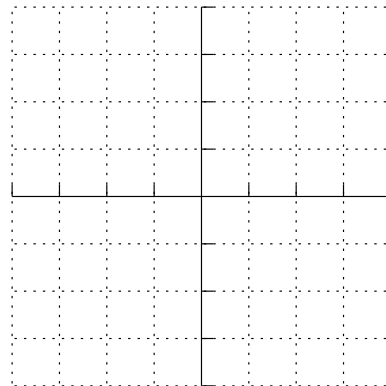
(c) $y = 3f(x/2)$



(d) $y = (f(x))^2$



(e) $y = 1/f(x)$



Assignment 10. (Due March 29) Suppose f is a function with the property that $f(a \cdot b) = f(a) + f(b)$, for all a and b in its domain.

(a) If $f(1)$ is defined, what is it?

(b) If $f(-1)$ is also defined, what is it?

(c) Show that f is even, that is, show $f(-x) = f(x)$, whenever both are defined.

(d) If $f(0)$ is defined, what is $f(x)$?

Assignment 11. (Due April 5) Let $f(x)$ be a increasing function. Show that $f(x)$ must be one-to-one. What can you say about $f^{-1}(x)$? Prove your claims.

Assignment 12. (Due April 12) You will not be able to solve $xe^x = 1$, but try to anyway. If you can do it, you'll get an A in the course. Your assignment is to prove that there is a single solution and that it is between 0 and 1. (You can see this using a graph, but we want a formal proof, using properties of functions like increasing and decreasing.) Use your calculator to estimate x to 3 decimal places.

Assignment 13. (Due April 19) Prove that $\log_2 3$ is an irrational number. Hints: Pretend that $\log_2 3$ is rational; what then? Let $m/n = \log_2 3$, where m and n are integers. Without loss of generality we may assume m and n have no common factors and are both positive. Why? From this derive a contradiction.