Lines and Linear Equations

Goals. Graph lines, find intercepts, find equation for a line, understand slope, find where two lines intersect. Also, given two points in the real plane $\mathbb{R}^2$ find the distance between them and the coordinates of their midpoint.

Examples. $y = x$, $y = 3x$, $y = -x/2$, ..., $y = mx$. Then $y = mx + b$ is called the **slope-intercept form**. But we could have $x = c$, where the slope is infinite or undefined. Finally $ax + by = c$ is called the **standard linear form**.

Let (a, b) and (p, q) be given points in $\mathbb{R}^2$.

Distance formula. The distance bewteen them is

$$d = \sqrt{(p-a)^2 + (q-b)^2}.$$

Midpoint formula. The coordinates of their mid-point is

$$\left(\frac{a+p}{2}, \frac{b+q}{2}\right).$$

Slope formula. The slope of the line through them, when defined, is

$$m = \frac{q-b}{p-q}.$$

Line. A formula for the line that passes through these two points is given by

$$y - b = m(x - a),$$

which can be easliy converted to the slope-intercept form.

Examples.

Fact. If the slopes of two lines are negative reciprocals then the lines are perpendicular.

Proof.

Intersection of two lines. Given two distinct lines that are not parallel, that is they have different slopes, they will meet at a point.

Examples.

Application. Find the distance between the line $y = x + 2$ and the point $(2, 7)$.

Step 0. Draw picture.

Step 1. Find equation for the line through $(2, 7)$ that is perpendicular to $y = x + 2$. Hint: the slope is $1/-1 = -1$.

Step 2. Find where this line and the given line intersect.

Step 3. Find distance between this intersection point and $(2, 7)$. Should get

$$\frac{3}{2}\sqrt{2} \approx 2.1213.$$

Exercises.

1.1 (Graphs) 15, 17, 19, 21, 27, 31. Label the intersecpts!
1.1 (Distances) 39, 41, 43, 47.
1.1 (Midpoints) 57, 61, 65.
1.3 (Slopes) 5, 6, 7, 9, 13, 17.
1.4 (More lines) 13, 19, 25, 35, 47, 53, 55, 57, 61, 67. (There are two correct answers to 67.)
2.1 (Even more lines) 1, 9, 23, 27, 53, 61, 93.
8.1 (Intersection points) 17, 19, 29. (Use either method.)

We will have a quiz on lines sometime next week.