Facts from Geometry
for Math 109

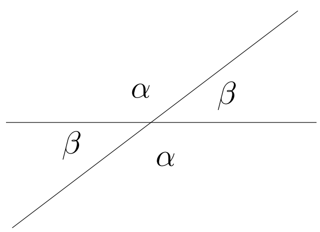
• Some commonly used Greek letters:

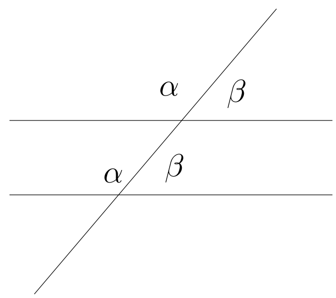
α	alpha
β	beta
γ	gamma
θ	theta
ϕ	phi
ψ	psi
π	pi

• Angles.

I will assume you are familiar with how to measure angles in degrees: 360^o is all they way around, a quarter of this is 90^o the so called **right angle**.

When two lines cross the opposite angles are equal and the adjacent angles add to 180^o. If a line crosses two parallel lines the corresponding angles are equal. See the two figures below.

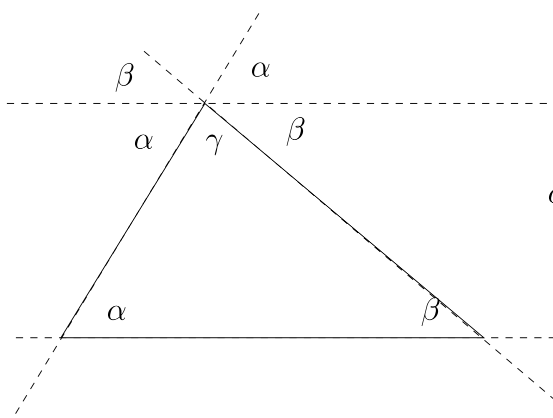




• Triangles.

Theorem: The sum of the interior angles of a triangle is always 180^o.

To see this apply the two facts about angles above to the figure below.

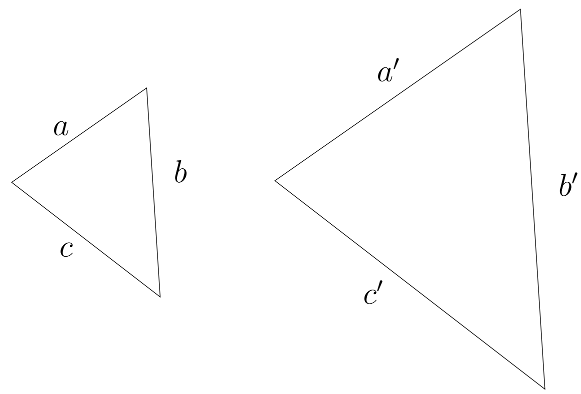


$$\alpha + \gamma + \beta = 180^o$$

QUESTION. What is the sum of the interior angles of a quadrilateral (a four sided figure)? What about a five sided figure? What about for an n-sided figure?

DEFINITION. Two triangles are **similar** if they have the same angles.

IMPORTANT THEOREM. For similar triangles the corresponding edges have equal ratios.

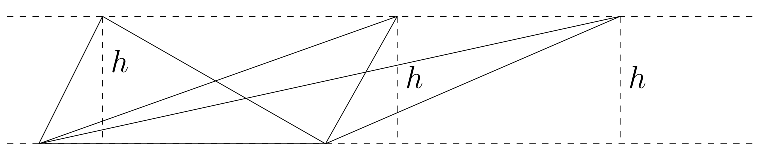


$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$

It follows that $\frac{a}{b} = \frac{a'}{b'}$ and so on.

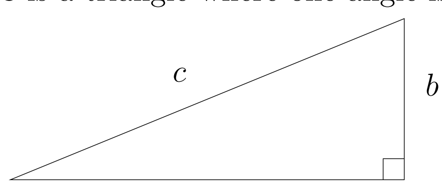
THEOREM. The area enclosed by a triangle is one half the base times the height. This works no matter which side is chosen as the base. In fact the vertex not on the base need not be directly above the base.

Notice that the areas of the three triangles in the figure below are equal.



- Special triangles.

A **right triangle** is a triangle where one angle is 90^o.



THE PYTHAGOREAN THEOREM (CIRCA 500 BC). For a right triangle

$$a^2 + b^2 = c^2$$

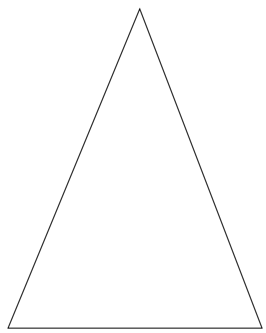
where c is the length of the hypotenuse (the side opposite the 90^o angle) and a and b are the lengths of the legs (the other two sides).

An **isosceles triangle** is a triangle where two of the angles are equal. It follows that two of the sides are equal.

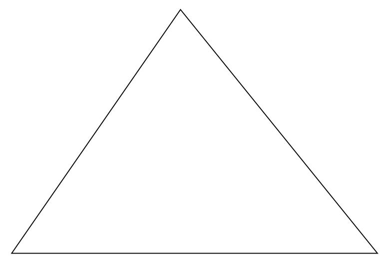
QUESTION. If an isosceles triangle has base 4 units and the other two sides are 3 units, what is its area? (*Answer*: $2\sqrt{5}$ units squared.)

An **equilateral triangle** is a triangle where all there angles and all three sides are equal.

QUESTION. What is the interior angle of an equilateral triangle? (60^o)



An isosceles triangle



An equilateral triangle

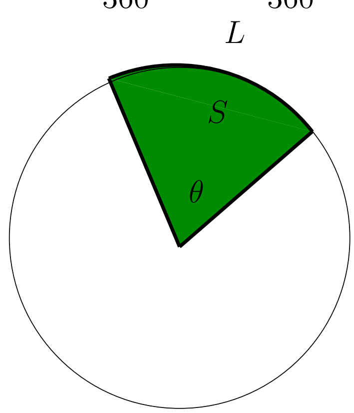
• Circles.

For a circle with radius r the circumference is given by $C = 2\pi r$ and the enclosed area is given by $A = \pi r^2$. Suppose two rays emanating from the center of a circle form an angle θ. Then the length of the arc along the circle between where the two rays cross the circle is

$$L = \frac{\theta}{360} \times C = \frac{\theta \pi r}{180}$$

This arc is called the arc **subtended** by θ. The area of the wedge shaped region (it is called a **sector**) formed by the rays and the arc is

$$S = \frac{\theta}{360} \times A = \frac{\theta \pi r^2}{360}$$



• Radians.

So far you should have seen everything we have covered in your high school geometry course. We now introduce the first new idea. It is a new way of measuring angles called **radians**.

The circle with radius one centered at the origin in the xy-plane is call the **unit circle**. Consider an angle θ determined by two rays meeting at the origin. Suppose the arc it subtends on the unit circle has length L. We shall use this length as a measure of the angle θ. For example a 90^o angle subtends an arc that goes one forth of the way along the unit circle. Its length is $2\pi/4 = \pi/2 \approx 1.57079632579$. So we say $\theta = \pi/2$ radians. You should check that 180^o is π radians, 45^o is $\pi/4$ radians, and that 30^o is $\pi/6$ radians. It is easy to deduce a formula for converting from degrees to radians.

$$\text{radians} = \frac{\text{degrees}}{360} \times 2\pi = \text{degrees} \times \frac{\pi}{180}$$

Using radians the formulas for arc lengths and sector areas become simpler.

$$L = \theta r \qquad S = \frac{1}{2}\theta r^2$$

Now we are ready to start trigonometry!