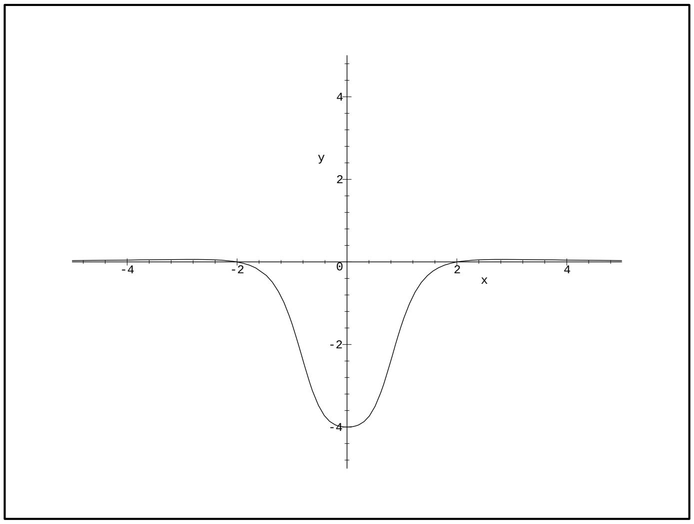
A Plotting Example with Maple

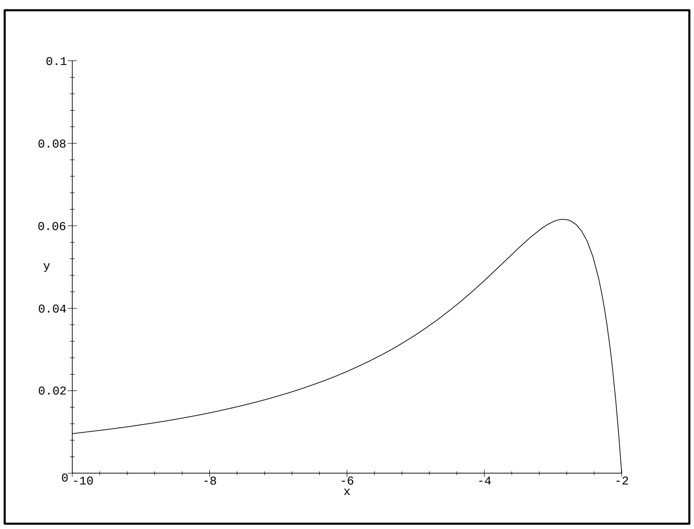
Plot the function f(x) = $\frac{x^2-4}{x^4+1}$. Label the extrema and inflection points.
STEP 1: We do a first plot to get a feel for the function

```
> plot((x^2-4)/(x^4+1),x=-5..5,y=-5..5,color=bl ack,thickness=2);
```



This looks different than our hand plot in class. We may need to plot it in sections. After some experimenting I came up with this.

```
> plot((x^2-4)/(x^4+1),x=-10..-2,y=0..0.1,color =black,thickness=2);
```



STEP 2: Find the extreme.

```
> diff((x^2-4)/(x^4+1),x);
>
```

$$2\,\frac{x}{x^4+1}-4\,\frac{(x^2-4)\,x^3}{(x^4+1)^2}$$

```
> simplify(");
>
```

$$-2\,\frac{x\,(x^4-1-8\,x^2)}{(x^4+1)^2}$$

So one zero of f'(x) is at x=0. For the others we solve $x^4-8\,x^2-1=0$. Let $a=x^2$.

Then $a=\frac{8+\sqrt{64+4}}{2}$ or $\frac{8-\sqrt{64+4}}{2}$. But the second is negative so x won't be real. Thus, x is plus or minus $\sqrt{4+\sqrt{17}}$.

```
> f := x ->(x^2-4)/(x^4+1):
```

> We make a function. This will reduce our typing.

```
> f(sqrt(4+sqrt(17))):
>
```

$$\frac{\sqrt{17}}{(4+\sqrt{17})^2+1}$$

Let's convert this to decimal form.

```
> evalf(");
>
```

$$.06155281281$$

STEP 3: We compute f"(x).

```
> simplify(diff(f(x),x$2));
>
```

$$2\,\frac{3\,x^8-12\,x^4+1-40\,x^6+24\,x^2}{(x^4+1)^3}$$

```
> fsolve(3*a^4-12*a^2+1-40*a^3+24*a,a,real);
>
```

$$-.8862623902,\ -.04094323555,\ .6762337382,\ 13.58430522$$

Two of these are negative. We only want the square roots of the positive ones.

```
> sqrt(.6762337382);
>
```

$$.8223343226$$

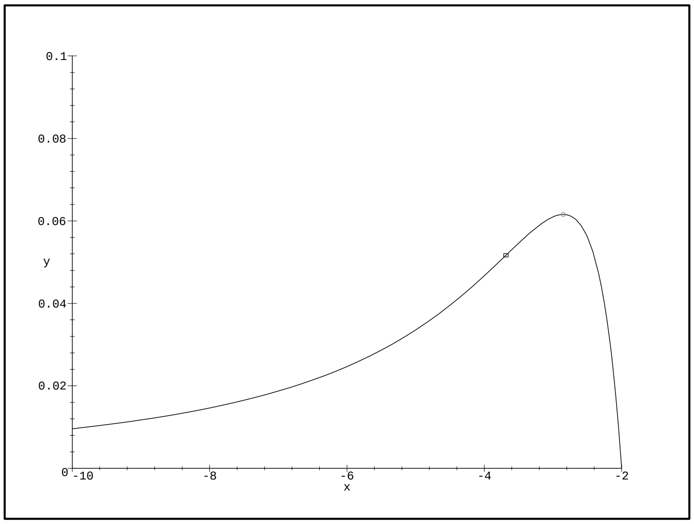
```
> sqrt(13.58430522);
>
```

$$3.685689246$$

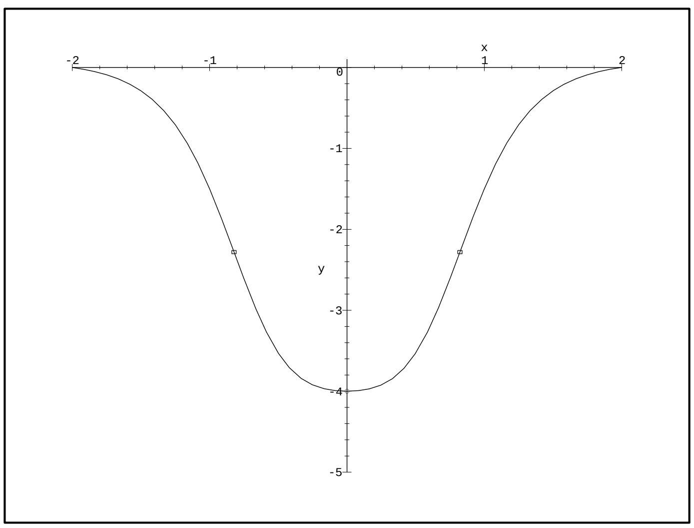
STEP 4: We put it all together a make a nice plot (in sections). I am going to use some fancy graph commands, but you can add some of the labels to your graph by hand.

```
> with(plottools):
> Section1:=plot(f(x), x=-10..-2, y=0..0.1, color=black,thickness=2):
> Section2:=plot(f(x), x=-2..2, y=-5..0.1, color=black,thickness=2):
> Section3:=plot(f(x), x=2..10, y=0..0.1, color=black,thickness=2):
> Max1:=point([-sqrt(4+sqrt(17)),f(sqrt(4+sqrt( 17)))],color=green,symbol=circle):
> Max2:=point([0,-4], color=green,symbol=circle):
> Max3:=point([sqrt(4+sqrt(17)),f(sqrt(4+sqrt(1 7)))], color=green,symbol=circle):
```

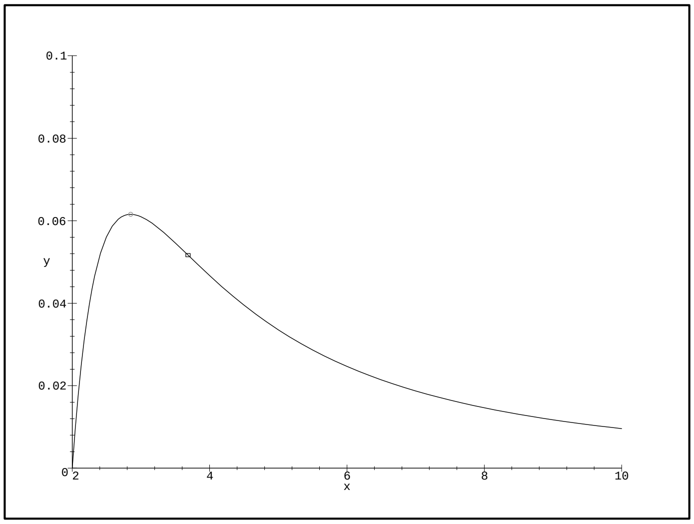
```
> Inflect1:=point([-3.685689246,f(-3.685689246) ], color=blue,symbol=box):
> Inflect2:=point([-.8223343226,f(-.8223343226) ], color=blue,symbol=box):
> Inflect3:=point([.8223343226,f(.8223343226)], color=blue,symbol=box):
> Inflect4:=point([3.685689246,f(3.685689246)], color=blue,symbol=box):
> plots[display](Section1,Max1,Inflect1);
```




```
> plots[display](Section2,Max2,Inflect2,Inflect 3);
```




```
> plots[display](Section3,Max3,Inflect4);
```



We are done. But notice that it looks as though the plots would not match up smoothly at -2 and +2. Explain why they do.