48. A model rocket is fired vertically upward from rest. Its acceleration for the first three seconds is $a(t) = 60t$, at which time its fuel is exhausted and it becomes a freely "falling" body. Fourteen seconds later, the rocket's parachute opens, and the (downward) velocity slows linearly to -18 ft/s in 5 s. The rocket then "floats" to the ground at that rate.
(a) Determine the position function s and the velocity function v (for all times t). Sketch the graphs of s and v.
(b) At what time does the rocket reach its maximum height, and what is that height?
(c) At what time does the rocket land?

Solution. This is how I read the problem. A rocket it fired upward from rest. **So, the initial velocity is 0. I'll assume it starts on the ground so the initial position is 0.**
For $0 < t < 3$, we have $a(t) = 60t$.
For $3 < t < 3 + 14 = 17$, we have "free fall". This means $a(t) = -g = -32$ ft/sec/sec.
For $17 < t < 17 + 5 = 22$, the velocity is linear, so $v(t) = at + b$, and we know $v(22) = -18$.
For $t > 22$ (until it lands), we have $v(t) = -18$, a constant.

I'm not going to worry about what $a(t)$ is at the end points of these intervals. It is clearly discontinuous. However $v(t)$ and $s(t)$ will be continuous. I'm going to put all this in a big table to better organize my thoughts. (I might not turn the table in; it just helps me to think.) Some of the minor calculations are left to the reader.

Time	Acceleration	Velocity	Position
$t \in (0, 3)$.	$a_1(t) = 60t$.	$v_1(t) = -30t^2 + C$ but $C = 0$	$s_1(t) = 10t^3 + C$ again $C = 0$
$t \in (3, 17)$.	$a_2(t) = -32$.	$v_2(t) = -32t + C$. Using $v_2(3) = v_1(3) = -270$ we get $C = 366$.	$s_2(t) = -16t^2 + 366t + C$. $s_2(3) = s_1(3) = 270$, gives $C = -684$.
$t \in (17, 22)$.	$a_3(t) = ??$ (From v_3 we get $a_3(t) = 32$)	$v_3(t) = at + b$. $v_3(17) = v_2(17) = -178$ & $v_3(22) = -18$. Thus, $v_2(t) = 32t - 722$.	$s_3(t) = 16t^2 - 722t + C$. Now $s_3(17) = s_2(17)$ gives $C = 8564$.
$t \in (22, ??)$.	$a_4(t) = 0$.	$v_4(t) = -18$.	$s_4(t) = -18t + C$ $s_4(22) = s_3(22) = 424$, gives $C = 820$.

I'm going to rewrite this in branched function notation.

$$a(t) = \begin{cases} 60t & \text{for} \quad t \in (0,3) \quad \text{Blast off!} \\ -32 & \text{for} \quad t \in (3,17) \quad \text{Free fall!} \\ 32 & \text{for} \quad t \in (17,22) \quad \text{Parachute opens, we're saved!} \\ 0 & \text{for} \quad t > 22 \quad \text{Until we land.} \end{cases}$$

$$v(t) = \begin{cases} 30t^2 & \text{for} \quad t \in [0,3] \\ -32t + 366 & \text{for} \quad t \in [3,17] \\ 32t - 722 & \text{for} \quad t \in [17,22] \\ -18 & \text{for} \quad t \geq 22 \text{ (until we land)} \end{cases}$$

$$s(t) = \begin{cases} 10t^3 & \text{for} \quad t \in [0,3] \\ -16t^2 + 366t - 684 & \text{for} \quad t \in [3,17] \\ 16t^2 - 722t + 8564 & \text{for} \quad t \in [17,22] \\ -18t + 820 & \text{for} \quad t \geq 22 \text{ (until we land)} \end{cases}$$

This answers part (a). Graphs are attached.

(b) The maximum height occurs when $v_2(t) = 0$ which is $t = 11.4375$ seconds. Then $s_2(11.4375) = 1409.0625$ feet.

(c) We solve for when $s_4(t) = 0$. This gives $t = 45\frac{5}{9}$ seconds after blast off.

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