Math 150 Quiz 3 Fall 2010

Name: Key

NO CALCULATORS

1. [5 points] What is the formal definition of the derivative?

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$, if the limit exists.

2. [10 points] Use the formal definition of the derivative to find $\left(\frac{1}{x+2}\right)'$.

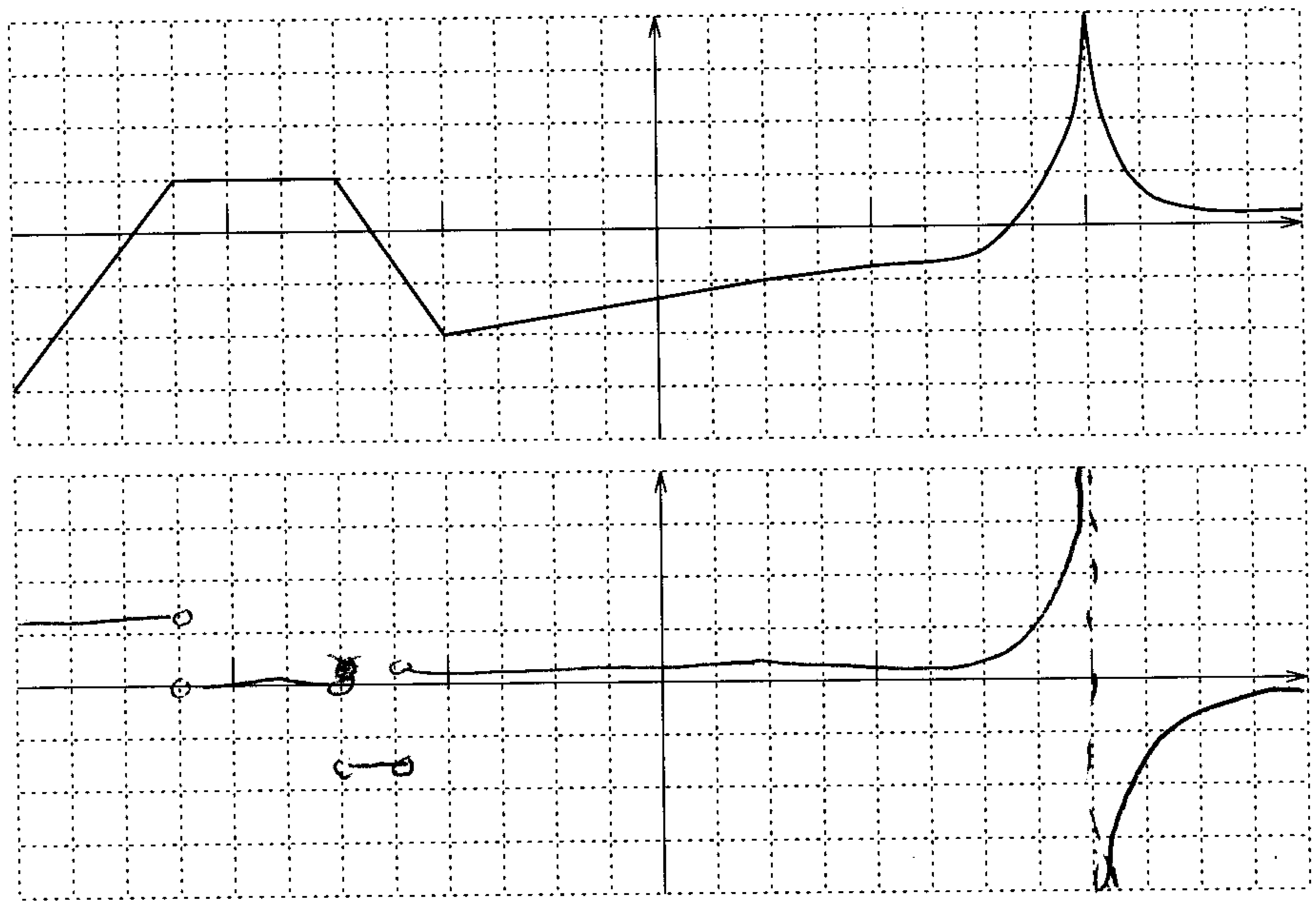
$$\left(\frac{1}{x+2}\right)' = \lim_{h \to 0} \frac{\frac{1}{(x+h)+2} - \frac{1}{x+2}}{h} = \lim_{h \to 0} \frac{1}{h} \frac{(x+2) - (x+h+2)}{(x+h+2)(x+2)}$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{-h}{(x+h+2)(x+2)} = \lim_{h \to 0} \frac{-1}{(x+h+2)(x+2)} =$$

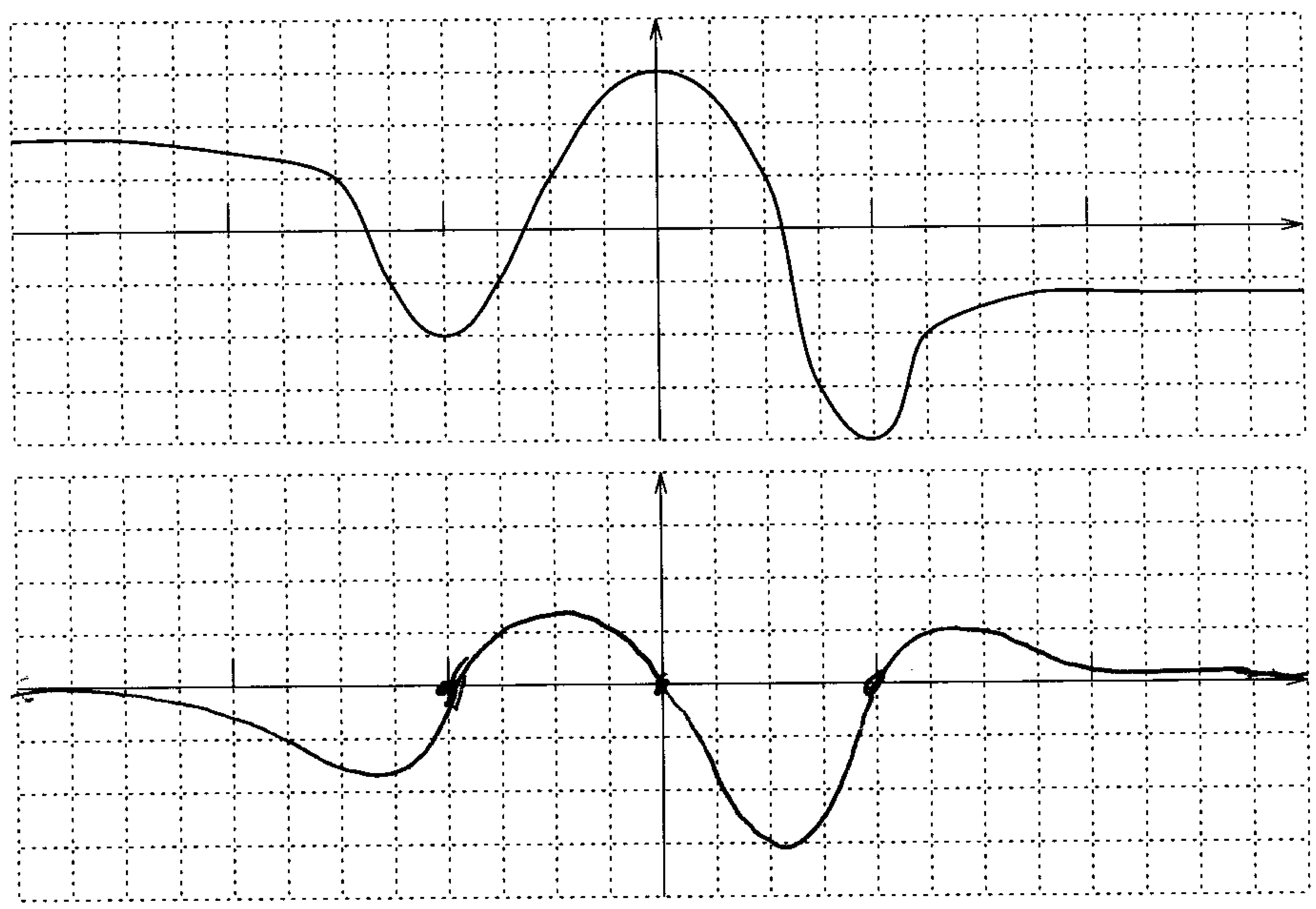
$$\frac{-1}{(x+0+2)(x+2)} = \boxed{\frac{-1}{(x+2)^2}}$$

3. [10 points] Below there are two graphs of functions with an empty graph grid below each. Draw the graph of the derivative of each in the grid below its graph.

(a)



(b)



4. [10 points] Find the derivatives below.

a. $\left(\frac{1}{x} + x^3 \sin x\right)'$ $= \frac{-1}{x^2} + (x^3)' \sin x + x^3 (\sin x)'$

$= \frac{-1}{x^2} + 3x^2 \sin x + x^3 \cos x$

b. $\left(x^3\sqrt{x} + \frac{x}{\csc x}\right)'$ $= (x^3)'\sqrt{x} + x^3(\sqrt{x})' + (x \sin x)'$

$= 3x^2\sqrt{x} + \frac{x^3}{2\sqrt{x}} + \sin x + x \cos x$

5. [5 points] Show that $(\tan x)' = \sec^2 x$.

$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{s'c - sc'}{c^2} = \frac{c^2 + s^2}{c^2} = \frac{1}{\cos^2 x} = \sec^2 x.$

6. [15 points] Find the limits below. Show each step you use.

valid for $x > 0$.

a. $\lim_{x\to\infty} \frac{\sqrt{3x^2+2}}{4x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{\frac{3x^2+2}{x^2}}}{4} = \lim_{x\to\infty} \frac{\sqrt{3+2/x^2}}{4} = \frac{\sqrt{3+0}}{4} = \frac{\sqrt{3}}{4}$

b. $\lim_{\theta\to 0} \frac{\tan 3\theta}{\theta} = \lim_{\theta\to 0} \frac{\sin 3\theta}{\theta} \cdot \frac{1}{\cos 3\theta}$

$= 3 \lim_{\theta\to 0} \frac{\sin 3\theta}{3\theta} \cdot \lim_{\theta\to 0} \frac{1}{\cos 3\theta} = 3 \cdot 1 \cdot \frac{1}{1} = 3.$

c. $\lim_{x\to\infty} \frac{x^2+x+3}{2x^2+5} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{1 + 1/x + 3/x^2}{2 + 5/x^2} = \frac{1+0+0}{2+0} = \frac{1}{2}$

Or invoke Theorem on Limits of Rational functions.