

Name: _____

Key

NON-GRAPHING CALCULATORS ALLOWED

1. [10 points] Suppose $x^2y + \tan(xy) = 7$. Find y' in terms of x and y .

$$(x^2y)' + (\tan(xy))' = 7'$$

$$(x^2)'y + x^2y' + \sec^2(xy)(xy)' = 0$$

$$2xy + x^2y' + \sec^2(xy)(x'y + xy') = 0$$

$$x^2y' + \sec^2(xy) \cdot y + \sec^2(xy) \cdot xy' = -2xy$$

$$[x^2 + \sec^2(xy)] y' = -2xy - y\sec^2(xy)$$

$$y' = \frac{-2xy - y\sec^2(xy)}{x^2 + x\sec^2(xy)}$$

2. [10 points] Let $z^2 = x^2 + y^2$. Suppose z , x , and y are functions of t . It is given that $dx/dt = 2$ and $dy/dt = 3$. Find dz/dt at the moment when $x = 5$ and $y = 12$.

$$2z z' = 2x x' + 2y y'$$

$$z' = \frac{x x' + y y'}{z}$$

$$= \frac{x x' + y y'}{\pm \sqrt{x^2 + y^2}} = \frac{5 \cdot 2 + 12 \cdot 3}{\pm \sqrt{25 + 144}}$$

$$= \pm \frac{46}{13} \approx \pm 3.538$$

(Prime means $\frac{d}{dt}$ here.)