

Name: Key**NON-GRAPHING CALCULATORS ALLOWED**

1. [10 points] Find the derivatives of the two functions below.

a.  $\cos^{-1}(3^x)$

$$\begin{aligned} (\cos^{-1}(3^x))' &= \frac{-1}{\sqrt{1-(3^x)^2}} (3^x)' \\ &= \frac{-3^x \cdot \ln 3}{\sqrt{1-3^{2x}}} \end{aligned}$$

b.  $\ln(\csc^2(3x)) = 2 \ln \frac{1}{\sin(3x)}$

$$= -2 \ln \sin 3x$$

$$(\quad)' = -2 \frac{1}{\sin 3x} (\sin 3x)'$$

$$= \frac{-6 \cos 3x}{\sin 3x} = -6 \cot 3x$$

2. [10 points] A water tank has the shape of an inverted circular cone with the radius of the top equal to 2 meters and a depth of 4 meters. If water is being pumped in at a rate of  $2 \text{ m}^3/\text{min}$ , what is the rate of change of the water level when the water is 3 meters deep? Given your answer is m/min rounded to the nearest 3rd decimal place.

See Example 3 on page 129.

3. [10 points] Find the following limits.

a.  $\lim_{x \rightarrow 0^+} \tan^{-1}\left(\frac{1}{x}\right)$

As  $x \rightarrow 0^+$ ,  $\frac{1}{x} \rightarrow \infty$ .

$$\lim_{x \rightarrow 0^+} \tan^{-1}\left(\frac{1}{x}\right) = \lim_{y \rightarrow \infty} \tan^{-1}(y) = \frac{\pi}{2}$$

b.  $\lim_{x \rightarrow 7} \ln|x-7|$

goes to  $-\infty$  from both sides, so limit is  $-\infty$

