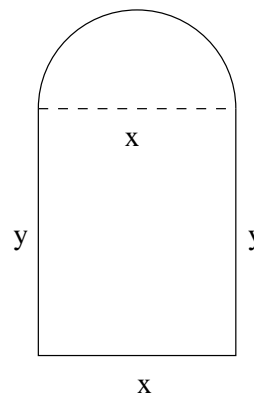


Name: _____

CALCULATORS ALLOWED

1. [10 points] A Norman window is constructed by adjoining a semi-circle to the top of an ordinary rectangular window. Find the dimensions of a Norman window of maximum area if the total perimeter is 16 feet.



The perimeter is 16 ft.

$$P = 2y + x + \frac{1}{2}(2\pi r) = 2y + x + \pi \left(\frac{x}{2}\right) = 2y + \left(1 + \frac{\pi}{2}\right)x = 16.$$

We solve for y .

$$y = \frac{16 - \left(\frac{2+\pi}{2}\right)x}{2} = 8 - \left(\frac{2+\pi}{4}\right)x.$$

We find an equation for area.

$$A = xy + \frac{1}{2}\pi r^2 = xy + \frac{1}{2}\pi \left(\frac{x}{2}\right)^2 = xy + \frac{\pi}{8}x^2.$$

Substitute in for y . Now

$$A = x \left[8 - \left(\frac{2+\pi}{4}\right)x \right] + \frac{\pi}{8}x^2 = 8x + \left(\frac{\pi}{8} - \frac{2+\pi}{4}\right)x^2 = 8x - \left(\frac{\pi+4}{8}\right)x^2.$$

Next compute dA/dx . Set it equal to zero and solve for x .

$$\frac{dA}{dx} = 8 - \left(\frac{\pi+4}{4}\right)x \stackrel{\text{set}}{=} 0.$$

$$\text{Therefore } x = 8 \cdot \frac{4}{\pi+4} = \frac{32}{\pi+4} \approx 4.48 \text{ ft.}$$

Next find y .

$$y = 8 - \frac{\pi+2}{4} \frac{32}{\pi+4} = 8 - 8 \cdot \left(\frac{\pi+2}{\pi+4}\right) = 8 \left(1 - \frac{\pi+2}{\pi+4}\right) = 8 \left(\frac{\pi+4}{\pi+4} - \frac{\pi+2}{\pi+4}\right) = \frac{16}{\pi+4} \approx 2.24 \text{ ft.}$$

2. [10 points] Let $xy^2 - 2\sin(xy) = 4$. Find y' as a function of x and y .

See Test 1.

3. [10 points] Sketch the graph of $y = e^{-x^2}$. Find the x coordinates of the two inflection points. Indicate where the graph is concave up and concave down.

See Test 2.