Math 150 Test 3 Fall 2010

Name: Key

NON-GRAPHING CALCULATORS ALLOWED

1. [5 points each] Compute the following integrals.

a. $\int \sin 3x \, dx$

$-\frac{1}{3}\cos(3x) + C$

b. $\int x^2 \cos(x^3)\, dx$ $\quad u = x^3.\ du = 3x^2 dx$

$= \frac{1}{3}\int \cos u \, du = \frac{1}{3}\sin u + C$

$= \frac{1}{3}\sin(x^3) + C$

c. $\int q\sqrt{q+3}\, dq$

$u = q+3.\quad du = dq$

$q = u - 3$

$\int (u-3)u^{\frac{1}{2}}\, du = \int u^{3/2} - 3u^{\frac{1}{2}} du$

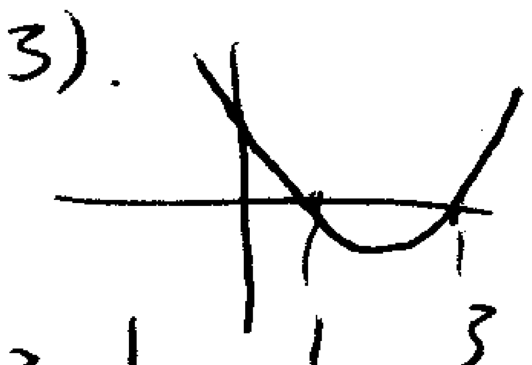
$= \frac{u^{5/2}}{\frac{5}{2}} - 3\frac{u^{3/2}}{\frac{3}{2}} + C = \frac{2}{5}(q+3)^{5/2} - 2(q+3)^{3/2} + C$

d. $\int_{-3}^{3} x^3 e^{x^2}\, dx$ (Think!)

The function odd, so the integral is zero.

e. $\int_0^3 |x^2 - 4x + 3|\, dx$

" $(x-1)(x-3)$.



$= \int_0^1 x^2 - 4x + 3\, dx - \int_1^3 x^2 - 4x + 3\, dx$

$= \left(\frac{x^3}{3} - 2x^2 + 3x\right)\Big|_0^1 - \left(\frac{x^3}{3} - 2x^2 - 3x\right)\Big|_1^3$

$= \left(\frac{1}{3} - 2 + 3\right) - \left[(9 - 18 - 9) - (\underbrace{\tfrac{1}{3} - 2 + 3}_{4/3})\right]$

$\frac{4}{3} + 18 + \frac{4}{3} = 18 + \frac{8}{3} = 20\frac{2}{3}$

f. $\int \tan\theta \, d\theta = \int \frac{\sin\theta}{\cos\theta} d\theta.$ $\quad u = \cos\theta.$ $du = -\sin\theta d\theta.$

$= -\int \frac{1}{u} du = -\ln|u| + C$

$= -\ln|\cos\theta| + C = \ln|\sec\theta| + C.$

2. [5 points] Let $g(x) = \int_0^{x^3} \sec^3(t^2)\,dt$. Find $\frac{dg}{dx}$. $= \sec^3((x^3)^2)\,(x^3)'$

$= \sec^3(x^6) \cdot 3x^2$

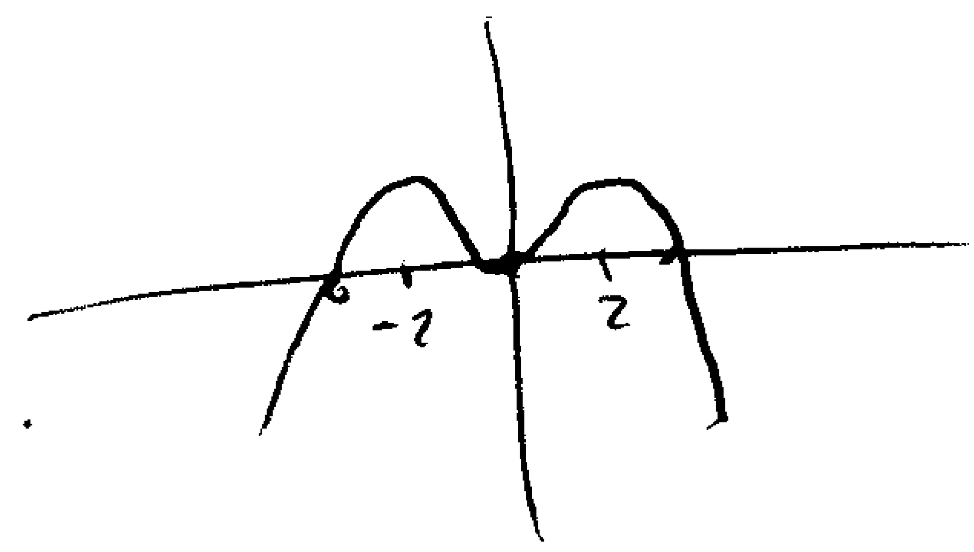
3. [10 points] Sketch the graph of $y = 8x^2 - x^4$. $= (8 - x^2)\,x^2 = (2\sqrt{2} - x)(2\sqrt{2} + x)\,x^2$.

a. Where are the zeros? $\pm 2\sqrt{2},\ 0$

b. What is the absolute maximum value and at which points does it occur?

c. What are the x-coordinates of the inflection points?

rough graph



b. $y' = 16x - 4x^3 = 0$

$4x(4 - x^2) = 0 \qquad x = 0, \pm 2$.

At $x = 2$ $\quad y = 8 \cdot 4 - 16 = 32 - 16 = 16$.

At $x = -2$, $y = 16$. $\quad$ Max is 16.

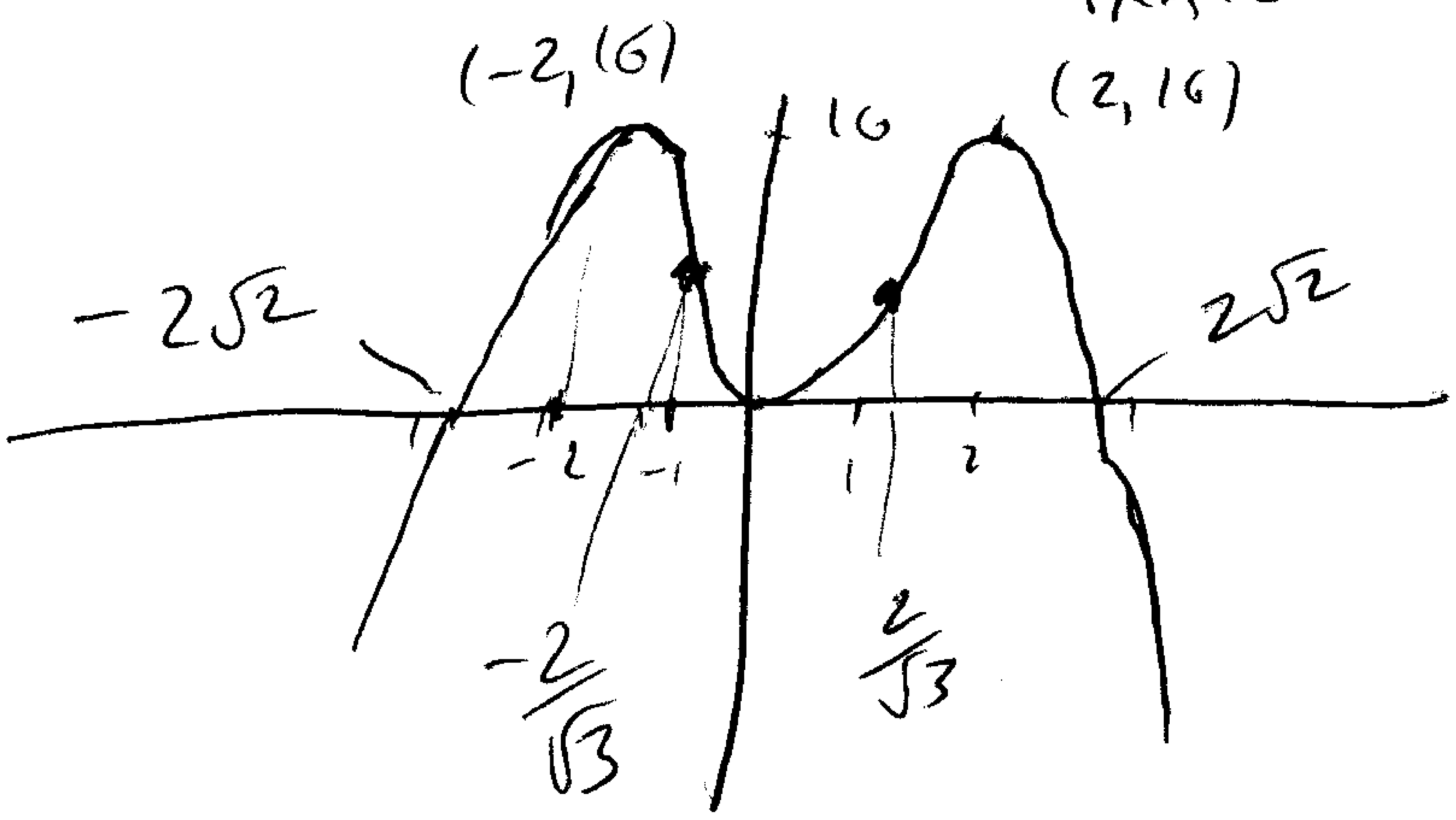
Occurs at $x = \pm 2$.

c. $y'' = 16 - 12x^2 = 0$

$4 - 3x^2 = 0$

$x^2 = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$. These are the x-coor. of the two inflection points.



4. [5 points] Recall that $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{2}$. Use this to compute $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^3}$.

$$= \lim_{n\to\infty} \frac{1}{n^3} \sum_{i=1}^{n} i^3 = \lim_{n\to\infty} \frac{1}{n^3} \frac{n^2(n+1)^2}{2} = \lim \frac{n^2(n^2+2n+1)}{2n^3}$$

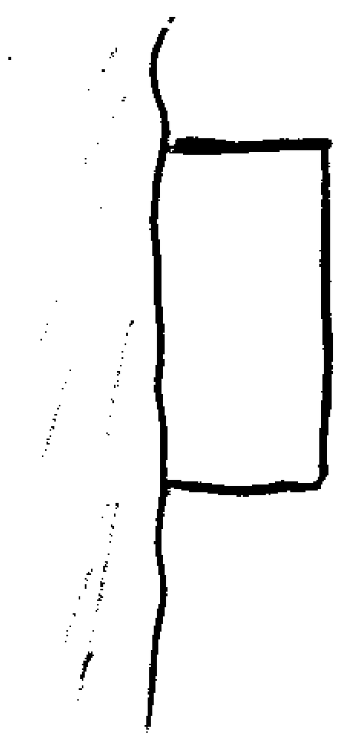
$$= \lim_{n\to\infty} \frac{n^4+2n^3+1}{2n^3} = \lim_{n\to\infty} \frac{n}{2} + 2 + \frac{1}{2n^3} = \infty + 2 + 0$$

$$= \infty.$$

[I meant to use n^4 not n^3. In that case you'd get $\frac{1}{2}$.]

5. [10 points] A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field with the largest area?

This was Example 1 on page 227.



6. [~~10~~ 20 points] Graph $f(x) = \int_0^x g(t)\,dt$, where $g(t)$ is given by the graph below and answer these questions. Indicate where your graph is concave up ad concave down.

What is $f(6)$? = total area = 1 + 1 + 1 + 4 = 7

What is $f'(2)$? = 1

What is $f''(2)$? = undef.

What is $f'(5)$? = 2

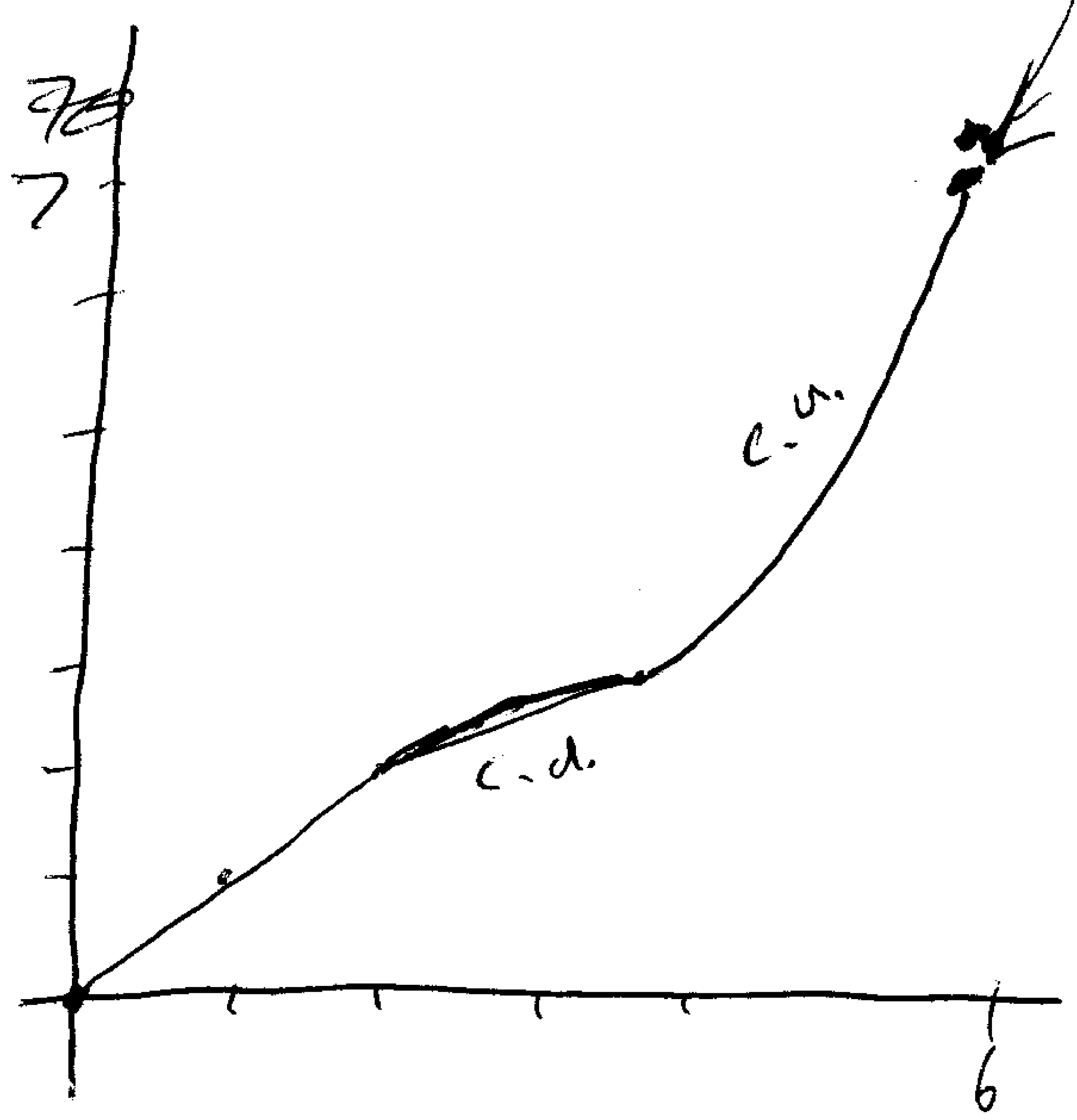
What is $f''(5)$? = slope = 2

$f(0) = 0$

$f(1) = 1$

$f(2) = 2$

$f(4) = 3$



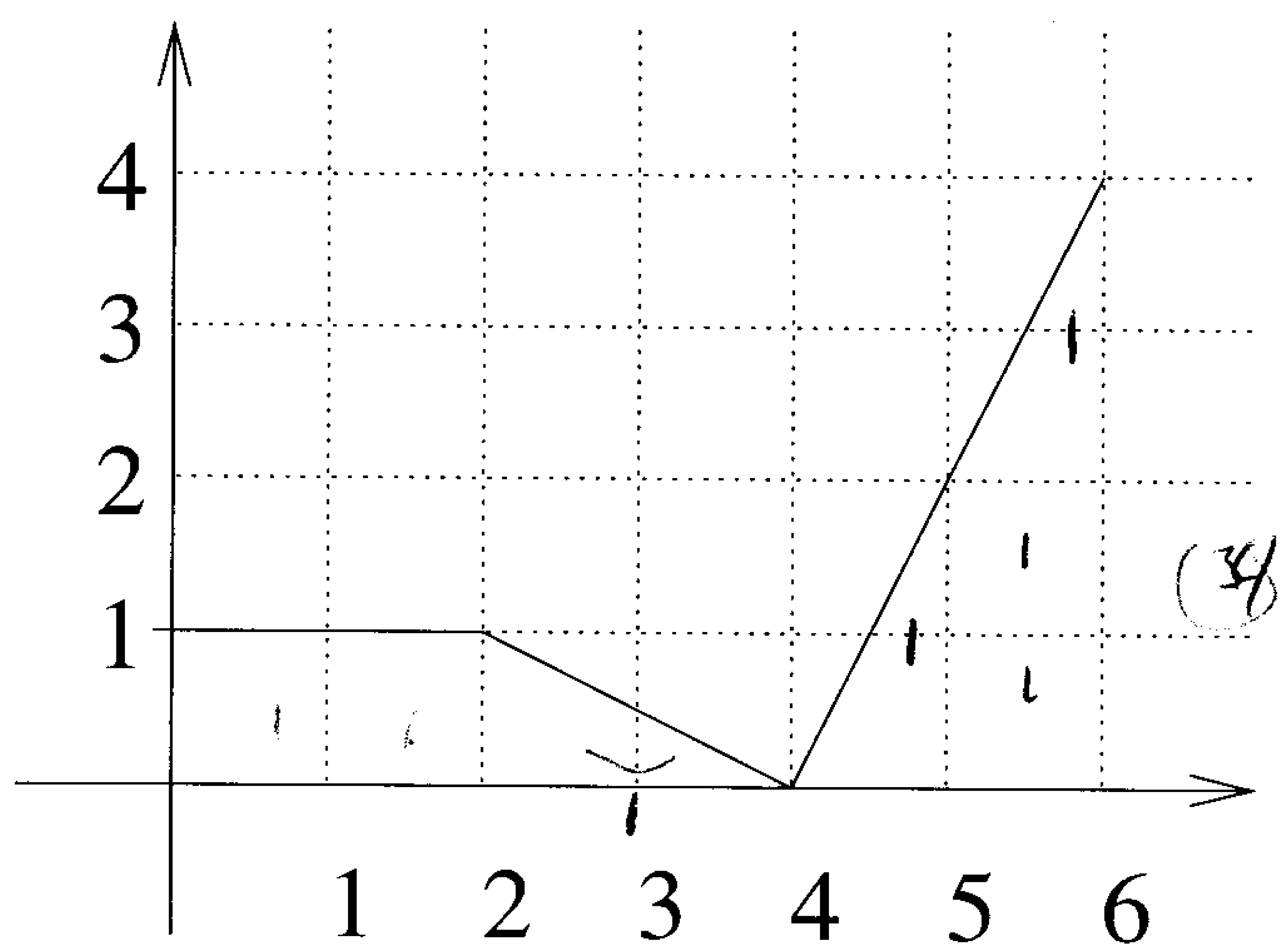


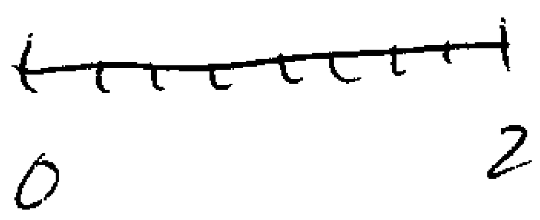
Figure 1: Graph of $g(t)$

7. [5 points] What is the average value of $f(x) = 4 - x^2$ over $[-2, 2]$?

$$\frac{1}{4}\int_{-2}^{2} 4 - x^2\,dx = \frac{1}{4}\left(4x - \frac{x^3}{3}\right)_{-2}^{2} = \frac{1}{4}\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right]$$

$$= \frac{1}{4}\left(16 - \frac{16}{3}\right) = \frac{16}{4}\left(1 - \frac{1}{3}\right) = 4 \cdot \frac{2}{3} = \frac{8}{3}.$$

8. [5 points] Let A be the area of the region under the graph of $y = e^{-x}$ from $x = 0$ to $x = 2$. Using right end points find an expression for A as a limit. DO NOT EVALUATE the limit.



$\Delta x = \frac{2}{n}$

$x_i = 0 + \frac{2}{n} i$

$$A_n = \sum_{i=1}^{n} e^{-x_i}\,\Delta x = \sum_{i=1}^{n} e^{-2\frac{i}{n}} \cdot \frac{2}{n}.$$

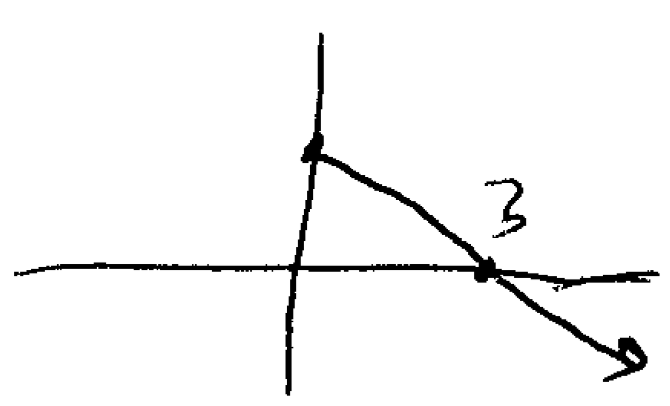
$$A = \lim_{n \to \infty} A_n.$$

[This was Example 3a on page 257.]

9. [10 points] Let $f(x) = \int_0^x (3 - x)e^x\,dx$ for $x \geq 0$. DO NOT ATTEMPT TO EVALUATE IT.

a. For what value of x will $f(x)$ be at its maximum value? Explain your reasoning.

Graph of $(3-x)e^x$



$f'(x) = (3-x)e^x. = 0$ at $x = 3$.

$\int_0^3 (3-x)e^x\,dx$ is positive. After 3, f will decrease. so max is at $x = 3$.

b. What is the x-coordinate of the inflection point of $f(x)$?

$$f'' = ((3-x)e^x)' = -e^x + (3-x)e^x = (2-x)e^x. = 0$$

when $x = 2$.

The infl. pt. occurs at $x = 2$.