Math 150 Test 2 Fall 2010

Name: _____

NON-GRAPHING CALCULATORS ALLOWED

1. [5 points each] Compute each of the following derivatives. You do not need to simplify your answers.

   a. $(\ln(x + \sin x))'$

   b. $\left((x^2 + \tan^{-1} 2x + 3^x)^{1/3}\right)'$

   c. $\left(\dfrac{x - \csc x}{x^2 + 3}\right)'$

   d. $\left(xe^{\sinh 2x}\right)'$

   e. $\left[\log_4(\sin^3(4x^6))\right]'$

   f. Let $y = (\ln x)^{x+1}$. Find $\dfrac{dy}{dx}$.

2. [10 points] Ship A and Ship B leave the same port at noon. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 p.m.? Give your answer to the nearest tenth km/h.

3. [20 points] a. State the Mean Value Theorem by filling in the blank spaces below.

The Mean Value Theorem: Let $[a, b]$ be a closed bounded interval. Suppose f is a function whose domain contains $[a, b]$ that satisfies the following two conditions.

1. f is _____ on $[a, b]$.

2. f is _____ on (a, b).

Then there exists a number $c \in (a, b)$ such that $f'(c) =$ _____ .

b. Draw a picture that illustrates the idea behind the Mean Value Theorem.

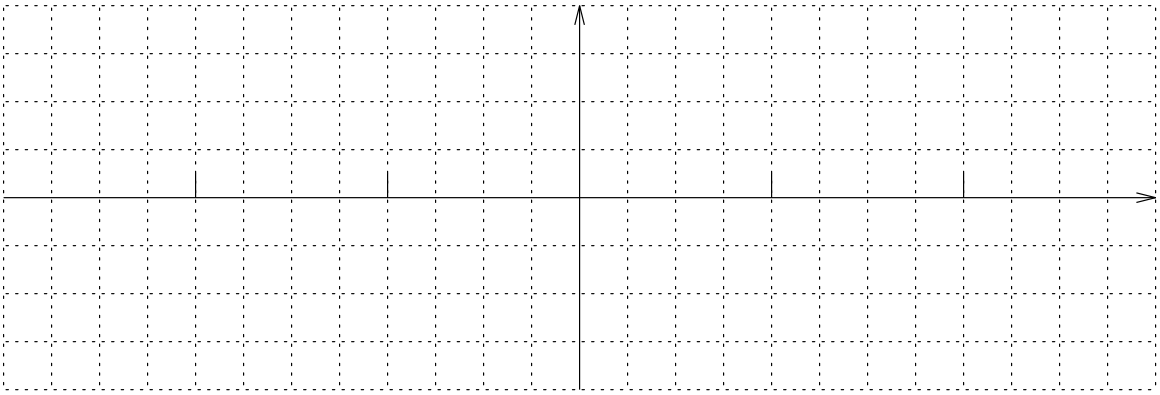
c. Let $f(x)$ be continuous and differentiable for all real numbers x. Suppose that $f(0) = -3$ and that $f'(x) \leq 5$ for all values of x. How large can $f(2)$ possibly be? Justify your answer.

d. Let $f(x) = 3x - 1 - \cos x$. Show that $f(x) = 0$ has exactly one real solution. Hint: first show there is a solution in the interval $(0, \pi)$.

4. [10 points] Sketch the graph of a function $y = f(x)$ that has the following properties. You may assume f is continuous and has continuous first and second derivatives.

   $f(0) = 3$
   For x in $(-\infty, -2)$ $f'(x) > 0$.
   For x in $(-2, 3)$ $f'(x) < 0$.
   For x in $(3, \infty)$ $f'(x) > 0$.
   For x in $(-\infty, -4)$ $f''(x) > 0$.
   For x in $(-4, 1)$ $f''(x) < 0$.
   For x in $(1, \infty)$ $f''(x) > 0$.

5. [10 points] Let $f(x)$ be a function. Below we show the **graph of the derivative**, $y = f'(x)$. Based on this graph answer these questions about the original function $f(x)$.
   a. On what intervals is $f(x)$ increasing?
   b. On what intervals is $f(x)$ decreasing?
   c. On what intervals is $f(x)$ concave up?
   d. On what intervals is $f(x)$ concave down?
   e. What are the x coordinates of the inflection points of $f(x)$.



6. [10 points] Sketch the graph of $y = e^{-x^2}$. Find the x coordinates of the two inflection points. Indicate where the graph is concave up and concave down.

7. [10 points] Sketch the graph of $y = \dfrac{x}{x^2+1}$. Find the values and locations of the absolute maximum and absolute minimum.

8. [10 BONUS points] Let $f(x)$ be a twice differential function that has a relative maximum at $x = c$ and has negative second derivative at $x = c$. Prove that $\arctan(f(x))$ must also have a relative maximum at $x = c$.