Math 150 Test 3 Fall 2011

Name: Key

NON-GRAPHING CALCULATORS ALLOWED

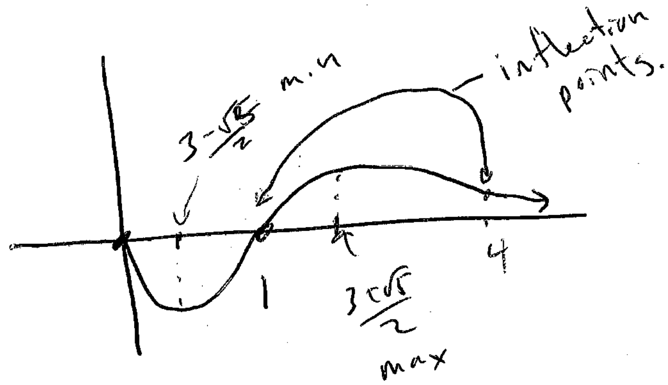
1. [15 points] Graph the function $f(x) = x(x-1)e^{-x}$ for $x \geqslant 0$.

   a. Label the zeros.

   b. Plug in some large numbers to see what it does for large values of x.

   c. Find and label the x coordinates of the local extrema.

   d. Find and label the x coordinates of the inflection points.



$f(10) = 0.004085994\ldots$

$f(20) = 0.0000007 83\ldots$

$f' = (x^2-x)'e^{-x} + (x^2-x)(e^{-x})' = (2x-1)e^{-x} - (x^2-x)e^{-x}$

$= (-x^2+3x-1)e^{-x}$

Thus $f'(x)=0$ when $x = \frac{-3 \pm \sqrt{9-4}}{-2} = \frac{3 \pm \sqrt{5}}{2}$

$f''(x) = (-x^2+3x-1)'e^{-x} + (-x^2+3x-1)(e^{-x})' =$

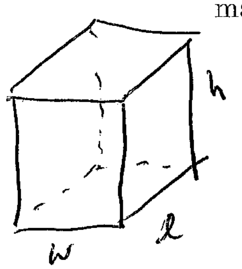
$(-2x+3)e^{-x} - (-x^2+3x-1)e^{-x} = (x^2-5x+4)e^{-x}$

$= (x-4)(x-1)e^{-x}$

Thus $f''(x) = 0$ at $x=1$ and $x=4$.

These are the x-coord.'s of the inflection pts.

2. [15 points] A box, with a top, is to have length twice its width and be made from 48 square feet of material. Find the dimensions that maximize the volume.



$l = 2w \qquad V = wlh$

$S.A. = 2wl + 2lh + 2wh = 48$

$4w^2 + 4wh + 2wh = 48$

$4w^2 + 6wh = 48$

$h = \frac{48 - 4w^2}{6w} = \frac{24 - 2w^2}{3w}$

$V = wlh = 2w^2h = 2w^2\left(\frac{24 - 2w^2}{3w}\right) = \frac{48}{3}w - \frac{4}{3}w^3$

$= 16w - \frac{4}{3}w^3$

$\frac{dV}{dw} = 16 - 4w^2 = 0$ when $w = 2$. Thus $l = 4$. $h = \frac{24 - 8}{6} = \frac{8}{3}$.

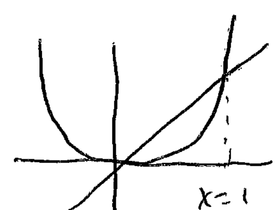
3. [15 points] Consider the area between the graphs of $y = x$ and $y = x^4$

.

a) Determine the area between the curves.

b) Write down an integral which gives the volume of the solid obtained by revolving this region around the x-axis. Do not evaluate.

c) Write down an integral for the volume of the solid obtained by revolving this area around the y-axis. Do not evaluate.



a) $\int_0^1 x - x^4 \, dx = \frac{1}{2}x^2 - \frac{1}{5}x^5 \Big|_0^1 = \frac{1}{2} - \frac{1}{5} = \frac{3}{10}$

b) Washers.

$\int_0^1 \pi(x)^2 - \pi(x^4)^2 \, dx$

b) shells

$\int_0^1 2\pi y(y^{\frac{1}{4}} - y) \, dy$

c) Shells

$\int_0^1 2\pi \underset{r}{x} \underset{h}{(x - x^4)} \, dx$

c) Washers

$\int_0^1 \pi(y^{\frac{1}{4}})^2 - \pi y^2 \, dy$

15

4. [~~25~~ 5 points] Find the folowing indefinite integrals. Clearly indicate any substitutions you use.

a. $\int \sin^2 x \cos x \, dx$ $= \int u^2 \, du = \frac{1}{3}u^3 + C = \frac{1}{3} \sin^3 x + C$

Let $u = \sin x$. Then $du = \cos x \, dx$

b. $\int 7x^3 + \frac{3}{x^5} + \pi \, dx$ $= \frac{7}{4} x^4 - \frac{3}{4} x^{-4} + \pi x + C$

c. $\int 3x\sqrt{3 - x^2} \, dx$ $= \frac{3}{-2} \int \sqrt{u} \, du = -\frac{3}{2} \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

$u^{\frac{1}{2}}$

$u = 3 - x^2.$

$du = -2x \, dx$

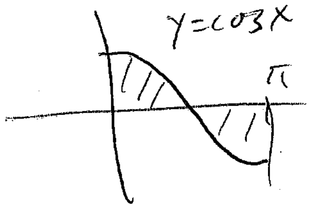
$= -(3 - x^2)^{3/2} + C$

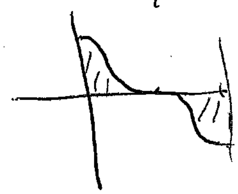
~~d. $\int dx$~~

~~e. $\int dx$~~

5. [20 points] Compute the folowing definite integrals. If you use a “trick” explain what you are doing. If the integral does not exist explain why.

a. $\int_0^{\pi} \cos^5 x\, dx \quad = 0$



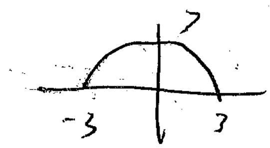


areas cancel.

$\int_0^{\pi} \cos x\, dx = 0$

b. $\int_0^{\pi/4} \sec^2 x\, dx \quad = \tan x \Big|_0^{\pi/4} = \tan(\frac{\pi}{4}) - \tan(0) = 1 - 0 = 1$

c. $\int_{-3}^{3} \sqrt{9 - x^2}\, dx$



$\frac{\pi r^2}{2} = \frac{9\pi}{2}$

d. $\int_0^{\pi} \sec x \tan x\, dx$

does not exist.

sec x tan x is not defined at $x = \frac{\pi}{2}$.

6. [20 points] The graph below gives the velocity $v(t)$ in meters per seconds of an object as a function of time t for $0 \leq t \leq 8$ seconds.
a. What is the object's position when $t = 8$ seconds? (Assume the object's position starts at zero.)
b. How much distance did the object travel in 8 seconds?
c. Sketch a graph of the position $s(t)$ in meters as a function of time in seconds.

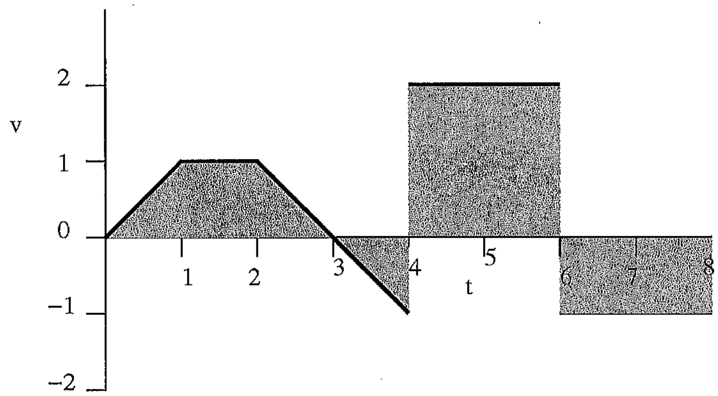


Figure 1: Velocity verses time

a, $s(t) = \int_0^8 v(t)\,dt =$ signed area $= \frac{1}{2} + 1 + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} + 4 - 2$

$3\frac{1}{2}$ meters.

b. dist. $= \int_0^8 |v(t)|\,dt = \underbrace{\frac{1}{2} + 1 + \frac{1}{2}}_{3} + \frac{1}{2} + 4 + 2 = 9\frac{1}{2}$ meters