Version 1

Math 150 Quiz 3 Fall 2011

Name: Key

NO CALCULATORS

1. [5 points] What is the formal definition of the derivative?

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \text{ when the limit exists.}$$

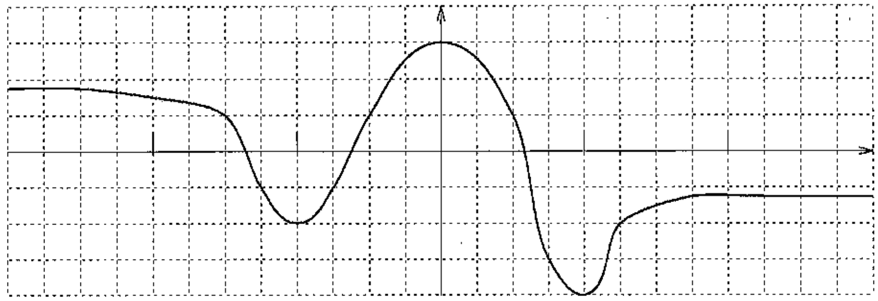
2. [10 points] Use the formal definition of the derivative to find $\left(\dfrac{1}{x-3}\right)'$.

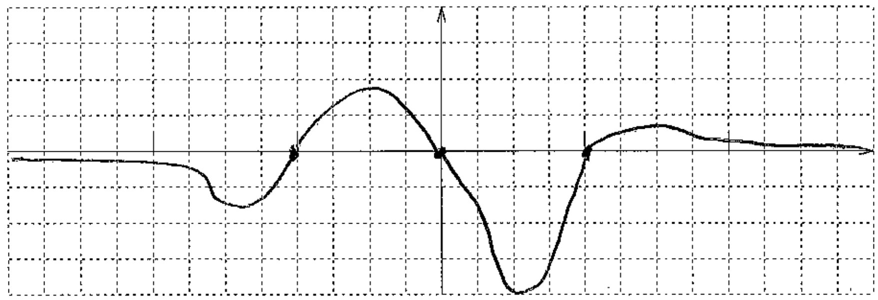
$$\left(\frac{1}{x-3}\right)' = \lim_{h \to 0} \frac{\frac{1}{x+h-3} - \frac{1}{x-3}}{h} = \lim_{h \to 0} \frac{\frac{(x-3)-(x+h-3)}{(x+h-3)(x-3)}}{h}$$

$$= \lim_{h \to 0} \frac{-h}{h(x+h-3)(x-3)} = \lim_{h \to 0} \frac{-1}{(x+h-3)(x-3)} = \frac{-1}{(x-3)^2}$$

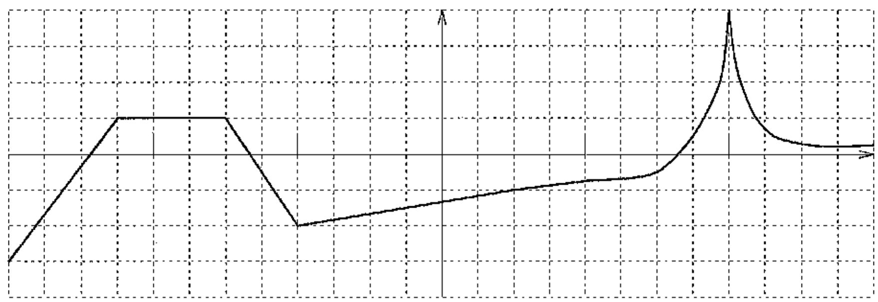
3. [10 points] Below there are two graphs of functions with an empty graph grid below each. Draw the graph of the derivative of each in the grid below its graph.

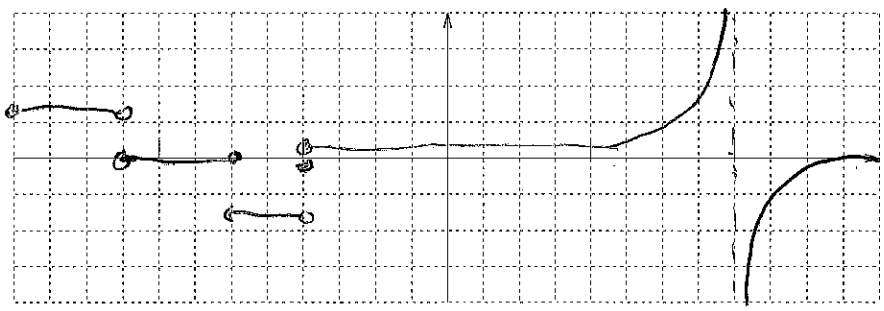
(a)





(b)





#3, continued.

e. $\lim_{v \to 4^+} \frac{4-v}{|4-v|}$

$\frac{4-v}{|4-v|} = -1$ for $v > 4$.

Thus the limit as $v \to 4^+$ is -1

f. $\lim_{x \to \infty} (\sqrt{x^2+4x+1} - x) \cdot \frac{\sqrt{\quad} + x}{\sqrt{\quad} + x}$

$$= \lim_{x \to \infty} \frac{x^2+4x+1 - x^2}{\sqrt{\quad} + x} = \lim_{x \to \infty} \frac{4x+1}{\sqrt{x^2+4x+1} + x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x \to \infty} \frac{4 + \frac{1}{x}}{\sqrt{1 + \frac{4}{x} + \frac{1}{x^2}} + 1} = \frac{4+0}{\sqrt{1+0+0}+1} = \frac{4}{2} = 2$$

g. $\lim_{x \to 1^-} \frac{x^2-9}{x^2+2x-3}$

$$= \lim_{x \to 1^-} \frac{(x-3)(x+3)}{(x+3)(x-1)}$$

didn't need to factor it!

$$\frac{-8}{0} = \pm\infty. \text{ which?}$$

We are coming into 1 from below. $x-3$ is neg and so is $x-1$. (and $x+3$, but it cancels anyway.)

Thus the function is + as $x \to 1$ from below. The limit is $+\infty$.

h. $\lim_{x \to \infty} \cos x^2$

Does not exist since cosine oscillates between -1 and 1.

4. [15 points - 5 points each] Find the derivatives below.

a. $\left(x^3 - 3x^2 - \frac{8}{x}\right)'$

$3x^2 - 6x + \frac{8}{x^2}$

b. $(\sqrt{x} + x\cos x)'$

$\frac{1}{2\sqrt{x}} + \cos x - x\sin x$

c. $(\sin^2 x + \cos^2 x)' = (1)' = 0$

5. [40 points - 5 points each] Find the limits below. Show each step you use.

a. $\lim_{x\to\pi^-} \csc x = \frac{1}{\sin x}$

$\sin x$ is + just below π, hence $\csc x$ will be + too. Thus limit is $+\infty$.

b. $\lim_{\theta\to 0} \frac{\tan 7\theta}{\theta} = \lim_{\theta\to 0} 7 \frac{\sin 7\theta}{7\theta} \frac{1}{\cos 7\theta}$

$= 7 \cdot 1 \cdot 1 = 7.$

c. $\lim_{x\to\infty} \frac{x^2+x+3}{2x^2+5} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{1 + \frac{1}{x} + \frac{3}{x^2}}{2 + \frac{5}{x^2}}$

$= \frac{1+0+0}{2+0} = \frac{1}{2}$

d. $\lim_{x\to\infty} \frac{\sqrt{3x^2+2}}{5x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{\sqrt{3 + \frac{2}{x^2}}}{5} = \frac{\sqrt{3+0}}{5} = \frac{\sqrt{3}}{5}$