Version 2

Math 150 Quiz 3 Fall 2011

Name: Key

NO CALCULATORS

1. [5 points] What is the formal definition of the derivative?

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$, when the limit exists.

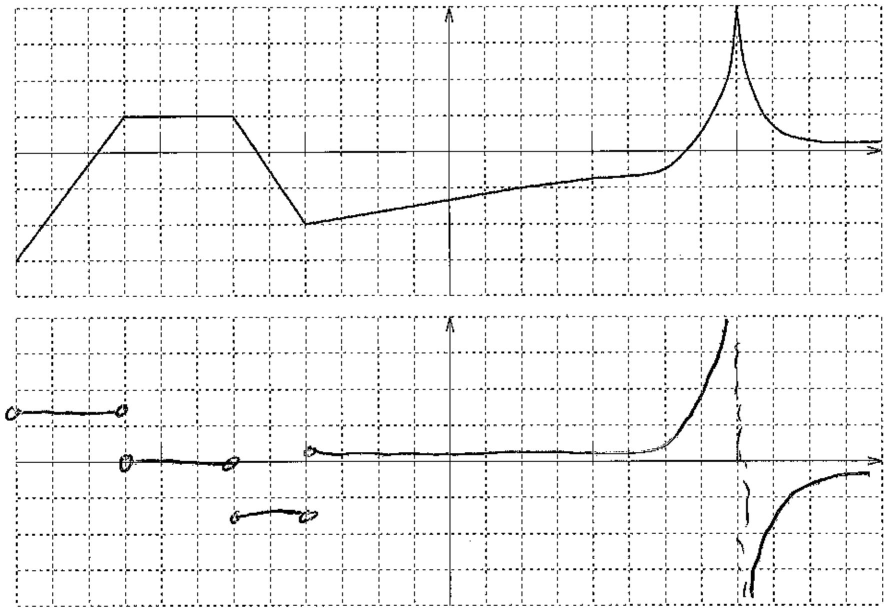
2. [10 points] Use the formal definition of the derivative to find $\left(\frac{1}{x+2}\right)'$.

$$\left(\frac{1}{x+2}\right)' = \lim_{h\to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} = \lim_{h\to 0} \frac{\frac{(x+2)-(x+h+2)}{(x+h+2)(x+2)}}{h}$$

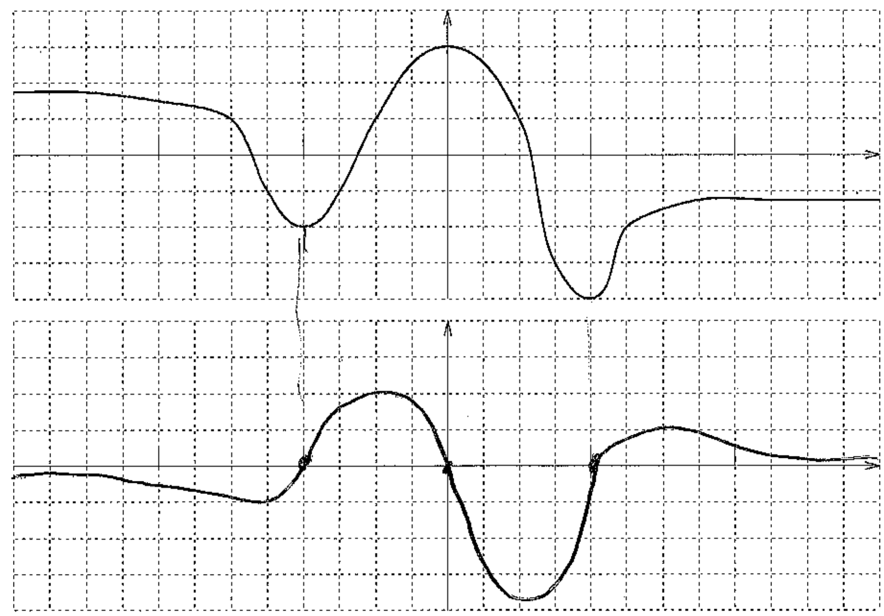
$$= \lim_{h\to 0} \frac{-h}{h(x+h+2)(x+2)} = \lim_{h\to 0} \frac{-1}{(x+h+2)(x+2)} = \frac{-1}{(x+0+2)(x+2)} = \frac{-1}{(x+2)^2}$$

3. [10 points] Below there are two graphs of functions with an empty graph grid below each. Draw the graph of the derivative of each in the grid below its graph.

(a)



(b)



4. [15 points - 5 points each] Find the derivatives below.

a. $\left(x^3 + 2x^2 - \frac{7}{x}\right)'$

$3x^2 + 4x + \frac{7}{x^2}$

b. $(\sqrt{x} + x\cos x)'$

$\frac{1}{2\sqrt{x}} + \cos x - x\sin x$

c. $(\sin^2 x + \cos^2 x)' = (1)' = 0$

5. [40 points - 5 points each] Find the limits below. Show each step you use.

a. $\lim_{x\to\infty} \frac{\sqrt{3x^2+2}}{4x}$ $\quad \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{\sqrt{3+\frac{2}{x^2}}}{4} = \frac{\sqrt{3+0}}{4} = \frac{\sqrt{3}}{4}$

b. $\lim_{\theta\to 0} \frac{\tan 3\theta}{\theta} = \lim_{\theta\to 0} 3\,\frac{\sin 3\theta}{3\theta}\,\frac{1}{\cos 3\theta}$

$= 3 \cdot 1 \cdot \frac{1}{1} = 3$

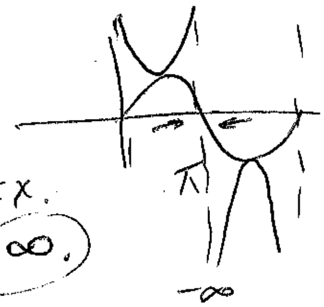
c. $\lim_{x\to\infty} \frac{x^2+x+3}{2x^2+5}$. $\quad \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{1+\frac{1}{x}+\frac{3}{x^2}}{2+\frac{5}{x^2}}$

$= \frac{1+0+0}{2+0} = \frac{1}{2}$

d. $\lim_{x\to\pi+} \csc x = \frac{1}{\sin x}$ $\quad +\infty$

Just above π $\sin x$ is $-$, hence so is $\csc x$. Thus limit is $-\infty$.



#3, continued.

e. $\lim_{x\to\infty} (\sqrt{x^2+4x+1} - x) \cdot \frac{\sqrt{x^2+4x+1} + x}{\sqrt{\quad} + x}$

$= \lim_{x\to\infty} \frac{x^2+4x+1-x^2}{\sqrt{\quad} + x} = \lim \frac{4x+1}{\sqrt{\quad} + x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{4+\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} + 1} = \frac{4+0}{\sqrt{1}+1} = 2$

f. $\lim_{v\to 4+} \frac{4-v}{|4-v|}$

For $v > 4$ $\frac{4-v}{|4-v|} = -1$,

Thus the limit $v \to 4$ from above is -1.

g. $\lim_{x\to 1^-} \frac{x^2-9}{x^2+2x-3} = \text{"}\frac{-8}{0}\text{"}$.

So the answer is $\pm\infty$.

Which?

For x just below 1,

x^2-9 is neg and so is x^2+2x-3.

Thus the function is + for x just below 1. Thus the limit is $+\infty$.

h. $\lim_{x\to\infty} \tan x^2$ d.n.e

tangent function ossillates between $\pm\infty$.