Math 150 Quiz 5 Fall 2010

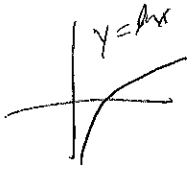
Name: Key

NON-GRAPHING CALCULATORS ALLOWED

1. [20 points] Find the limits below.

a. $\lim_{x\to 0^+} \ln x$

$= -\infty$



b. $\lim_{x\to\infty} \tan^{-1} e^x$ Let $y = e^x$. As $x \to \infty$, $y \to \infty$

$= \lim_{y\to\infty} \tan^{-1} y = \frac{\pi}{2}$

c. $\lim_{x\to\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$ $\frac{e^{-x}}{e^{-x}}$

$= \lim_{x\to\infty} \frac{1 - e^{-2x}}{1 + e^{-2x}} = \frac{1-0}{1+0} = 1$

d. $\lim_{x\to 0} \frac{x}{x + \tan x}$ $\frac{\frac{1}{x}}{\frac{1}{x}}$

$\lim_{x\to 0} \frac{1}{1 + \frac{\tan x}{x}} = \lim_{x\to 0} \frac{1}{1 + \frac{\sin x}{x}\cdot\frac{1}{\cos x}}$

$= \frac{1}{1 + 1\cdot\frac{1}{1}} = \frac{1}{1+1} = \frac{1}{2}$

2. [20 points] Find the derivatives of the functions below.

a. $\ln e^{x^4} = xy$.

$(xy)' = 4x^3$

b. $(\cot \ln x)'$

$-\csc^2(\ln x)(\ln x)' = -\frac{\csc^2 \ln x}{x}$

$= \frac{-1}{x \sin^2(\ln x)}$

c. $\left(\frac{3^x + x^2}{\log_7 x}\right)' =$

$\frac{(3^x + x^2)' \cdot \log_7 x - (3^x + x^2)(\log_7 x)'}{(\log_7 x)^2}$

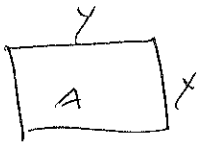
$= \frac{(3^x \ln 3 + 2x)\log_7 x - \frac{3^x + x^2}{x \ln 7}}{(\log_7 x)^2}$

d. $\left(e^{\tan(x^2)}\right)' = e^{\tan x^2} \cdot (\tan(x^4))'$

$= e^{\tan x^2} \sec^2(x^2)(x^2)'$

$= 2x \sec^2(x^2) e^{\tan(x^4)}$

3. [10 points] A screen saver displays the outline of a 3 cm by 2 cm rectangle and then expands the rectangle in such a way that the 2 cm side is expanding at the rate of 4 cm/sec and the proportions of the rectangle never change. How fast is the area of the rectangle increasing when its dimensions are 12 cm by 8 cm?



$A = xy$. $\quad \frac{x}{y} = \frac{2}{3} \Rightarrow y = \frac{3}{2}x$. $\quad \frac{dx}{dt} = 4$

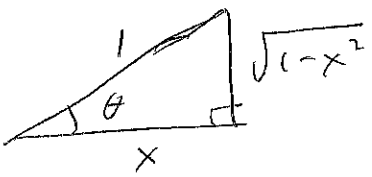
$A = \frac{3}{2}x^2$

$\frac{dA}{dt} = \frac{3}{2} \cdot 2x \frac{dx}{dt} = 3(8) \cdot 4 = 96 \text{ cm}^2/\text{sec}$

4. [10 points] Derive the formula for $(\cos^{-1}(x))'$. Express your final formula in algebraic form.

$\theta = \cos^{-1} x$

$\cos\theta = x$



$-\sin\theta \cdot \theta' = 1$

$\theta' = \frac{-1}{\sin\theta}$

$$\boxed{(\cos^{-1} x)' = \frac{-1}{\sqrt{1-x^2}}}$$