Math 150 Quiz ~~6~~ 7 Fall 2011

Name: Key

NON-GRAPHING CALCULATORS ALLOWED

1. [10 points] Find the derivatives of the functions below.

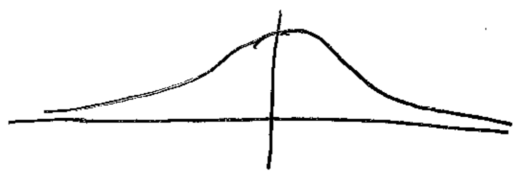
a. x^{7x}

$$y = x^{7x}$$
$$\ln y = 7x \ln x$$
$$\frac{y'}{y} = 7 \ln x + 7x \cdot \frac{1}{x}$$
$$y' = y(7 \ln x + 7)$$
$$(x^{7x})' = 7(x^{7x})(1 + \ln x)$$

b. $\sin^{-1}(\sec x)$

$$\frac{1}{\sqrt{1 - \sec^2 x}} \sec x \tan x$$

2. [10 points] Sketch the graph of $y = e^{-x^2}$. Find the x coordinates of the two inflection points. Indicate where the graph is concave up and concave down.



$$f' = -2x e^{-x^2}$$
$$f'' = -2e^{-x^2} + 4x^2 e^{-x^2} = 2(2x^2 - 1)e^{-x^2}$$

$f'' = 0$ when $x = \pm\sqrt{\frac{1}{2}}$. These are the inflection points.

3. [20 points] a. State the Mean Value Theorem by filling in the blank spaces below.

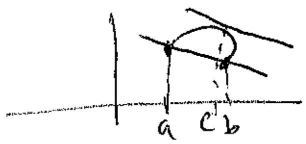
The Mean Value Theorem: Let $[a, b]$ be a closed bounded interval. Suppose f is a function whose domain contains $[a, b]$ that satisfies the following two conditions.

1. f is continuous on $[a, b]$.

2. f is differentiable on (a, b).

Then there exists a number $c \in (a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$

b. Draw a picture that illustrates the idea behind the Mean Value Theorem.



c. Let $f(x)$ be continuous and differentiable for all real numbers x. Suppose that $f(0) = -3$ and that $f'(x) \leq 5$ for all values of x. How large can $f(2)$ possibly be? Justify your answer.

$$\frac{f(2)-f(0)}{2-0} = f'(c) \leq 5$$

for some $c \in (0, 2)$

$$\frac{f(2)+3}{2} \leq 5$$

$$f(2) \leq 10-3=7.$$

If you are going never more than 5 mph, then in 2 hours, you can go at most 10 miles!

d. Let $f(x) = 3x - 1 - \cos x$. Show that $f(x) = 0$ has exactly one real solution. Hint: first show there is a solution in the interval $(0, \pi)$.

$f(0) = -1 - \cos(0) = -1-1 = -2 < 0$

$f(\pi) = 3\pi - 1 + 1 = 3\pi > 0.$

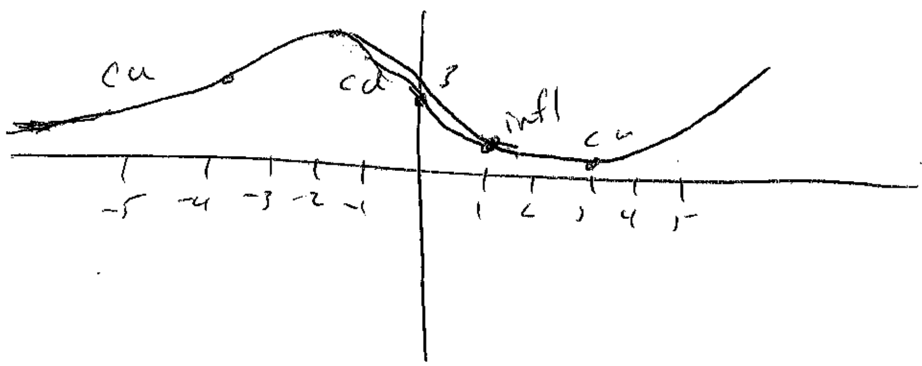
By the IVT $f(x)=0$ at least once in $(0, \pi)$.

$f'(x) = 3 + \sin x$, which can never be zero, $2 \leq f'(x) \leq 4$.

If $f(x)$ had a 2nd zero, then by Rolle's Thm $\exists$ an x in between them where $f'(x)=0$. But this can never happen.

4. [10 points] Sketch the graph of a function $y = f(x)$ that has the following properties. You may assume f is continuous and has continuous first and second derivatives.

$f(0) = 3$
For x in $(-\infty, -2)$ $f'(x) > 0$.
For x in $(-2, 3)$ $f'(x) < 0$.
For x in $(3, \infty)$ $f'(x) > 0$.
For x in $(-\infty, -4)$ $f''(x) > 0$.
For x in $(-4, 1)$ $f''(x) < 0$.
For x in $(1, \infty)$ $f''(x) > 0$.



5. [10 points] Let $f(x) = \frac{x}{x^2-5x+6}$. Sketch the graph of $y = f(x)$. Label the vertical asymptotes and indicate the horizontal asymptote. Find the x coordinates of the two local extrema and mark where they are on your graph. You'll be able to see which one is a local minimum and which one is a local maximum so you needn't bother with the Second Derivative test.

$$f(x) = \frac{x}{(x-3)(x-2)}$$

$$f' = \frac{x^2-5x+6 - x(2x-5)}{(\quad)^2}$$

$$= \frac{-\cancel{2}x^2+6}{(\quad)^2} = 0$$

$$x^2 = 6$$

$$x = \pm\sqrt{6}$$