Math 150 Test 1 Fall 2011

Name: ______________________

CALCULATORS ALLOWED

1. [5 points] Use the formal definition of a derivative to find $(\sqrt{x})'$.

See Example 3 on page 85.

2. [5 points] Let $f(x) = x^2 - 2\sqrt{x}$. Find the equation in slope-intercept form of the line tangent to the graph of $y = f(x)$ at the point (4,12).

$$f'(x) = 2x - 2\frac{1}{2\sqrt{x}} = 2x - \frac{1}{\sqrt{x}}$$

$$f'(4) = 8 - \frac{1}{2} = 7.5$$

$$y - 12 = 7.5(x - 4)$$

$$y = 7.5x - 30 + 12$$

$$\boxed{y = 7.5x - 18}$$

3. [20 points] Find the following limits. Show the steps you are using. Do not just plug in numbers.

a. $\lim_{x\to 0} \frac{\sin 3x}{\sin 5x}$ $= \lim_{x\to 0} \frac{\sin 3x}{x} \frac{x}{\sin 5x}$

$= \lim_{x\to 0} 3\frac{\sin 3x}{3x} \lim_{x\to 0} \frac{5x}{5\sin 5x}$

$= \frac{3}{5} \lim_{3x\to 0} \frac{\sin 3x}{3x} \lim_{5x\to 0} \frac{5x}{\sin 5x}$

$= \frac{3}{5} \cdot 1 \cdot 1 = \frac{3}{5}$

b. $\lim_{x\to 0} \frac{x}{x+\tan x}$ $\frac{0}{0}$ $\frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to 0} \frac{1}{1+\frac{\tan x}{x}}$

$\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x}$

$= 1 \cdot 1 = 1$

$= \frac{1}{1+1} = \frac{1}{2}$

c. $\lim_{x\to 3} \frac{x^2+x-12}{x^2-2x-3}$

$\frac{9+3-12}{9-6-3} = \frac{0}{0}$ factor.

$\lim_{x\to 3} \frac{(x+4)(x-3)}{(x+1)(x-3)} =$

$\lim_{x\to 3} \frac{x+4}{x+1} = \frac{3+4}{3+1} = \frac{7}{4}$

$\frac{7}{4}$

d. $\lim_{x\to\infty} \frac{\sqrt{4x^2+x}-2x}{1}$ $\frac{\sqrt{4x^2+x}+2x}{\sqrt{}+2x}$

$= \lim_{x\to\infty} \frac{(\cancel{4x^2}+x)-\cancel{4x^2}}{\sqrt{4x^2+x}+2x}$ $\frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{1}{\sqrt{\frac{4x^2+x}{x^2}}+2} = \lim_{x\to 0} \frac{1}{\sqrt{4+\frac{1}{x}}+2}$

$= \frac{1}{\sqrt{4+0}+2} = \frac{1}{2+2} = \frac{1}{4}$

4. [30 points] Find the following derivatives.

a. $(x^2 \sin x)'$

$$= 2x \sin x + x^2 \cos x$$

b. $(\sqrt{x} \cdot \cos x)'$

minus

$$= \frac{1}{2\sqrt{x}} \cos x \cancel{+} \sqrt{x} \sin x$$

c. $\left(\frac{7x+1}{x+3}\right)'$

$$= \frac{(7x+1)'(x+3) - (7x+1)(x+3)'}{(x+3)^2}$$

$$= \frac{7(x+3) - (7x+1)}{(x+3)^2} = \frac{2}{(x+3)^2}$$

d. $(x^4 - 2x^2 + 3\tan x)'$

$$= 4x^3 - 4x + 3\sec^2 x$$

e. $(\sin^3 x)'$

$$= 3 \sin^2(x) \cdot (\sin x)'$$

$$= 3 \sin^2(x) \cos(x)$$

f. $(\cot 3x)'$

$$-\csc^2(3x)\,(3x)'$$

$$= -3\csc^2(3x).$$

5. [10 points] Suppose the position of a object is given, in meters, by $s = t^2 - 8t + 18$, where t is time is seconds.

a. What is the average velocity over the time interval $3 \le t \le 4$?

$$\frac{s(4)-s(3)}{4-3} = \frac{-1}{1} = -1 \text{ m/s}$$

$$s(4) = 16 - 32 + 18$$
$$- \quad s(3) = 9 - 24 + 18$$
$$-16 - (-15) = -1$$

b. What is the instantaneous velocity at $t = 4$?

$$s'(t) = 2t - 8 \qquad s'(4) = 8 - 8 = 0 \text{ m/s}$$

6. [10 points] Let $xy^2 - 5xy = 6$. Find y' as a function of x and y.

Apply $\frac{d}{dx}$ to both sides.

$$(x)'y^2 + x(y^2)' - 5(x)'y - 5x(y)' = (6)'$$

$$y^2 + 2xyy' - 5y - 5xy' = 0$$

$$(2xy - 5x)y' = -y^2 + 5y$$

$$y' = \frac{-y^2+5y}{2xy-5x} = \frac{y(5-y)}{x(2y-5)}$$

7. [10 points] Let $f(x)$ and $g(x)$ be differentiable functions. Prove that

$$(f(x)+g(x))' = f'(x)+g'(x).$$

$$(f(x)+g(x))' = \lim_{h\to 0} \frac{[f(x+h)+g(x+h)] - [f(x)+g(x)]}{h} =$$

$$\lim_{h\to 0} \frac{[f(x+h)-f(x)] + [g(x+h)-g(x)]}{h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h}$$

$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = f'(x)+g'(x).$$

8. [10 points] For what value of x in $[0,\pi]$ does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent line? That is, when is the slope zero? Assume x is in radians.

Need $f'(x) = 0$.

$$f'(x) = 1 + 2\cos x = 0$$

$$\cos x = -\frac{1}{2}$$

$$x = \frac{3\pi}{2}$$