Math 150 Test 1 Fall 2011

Name: Key

CALCULATORS ALLOWED

1. [5 points] Use the formal definition of a derivative to find $(\sqrt{x})'$.

See Example 3 on page 85.

2. [5 points] Let $f(x) = x^2 + \sqrt{x}$. Find the equation in slope-intercept form of the line tangent to the graph of $y = f(x)$ at the point (4,18).

$f'(x) = 2x + \frac{1}{2\sqrt{x}}$ $\quad f'(4) = 8 + \frac{1}{2 \cdot 2} = 8.25$

$$y - 18 = 8.25(x-4)$$

$$y = 8.25x - 33 + 18$$

$$\boxed{y = 8.25x - 15}$$

3. [20 points] Find the following limits. Show the steps you are using. Do not just plug in numbers.

a. $\lim_{x\to 0} \frac{\sin 2x}{\sin 4x}$

$$= \lim_{x\to 0} \frac{\sin 2x}{x} \cdot \frac{x}{\sin 4x}$$

$$= \lim_{x\to 0} \frac{2\sin 2x}{2x} \cdot \lim_{x\to 0} \frac{4x}{4\sin 4x}$$

$$= \frac{2}{4} \lim_{2x\to 0} \frac{\sin 2x}{2x} \cdot \lim_{4x\to 0} \frac{4x}{\sin 4x}$$

$$= \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$$

b. $\lim_{x\to 3} \frac{x^2+x-12}{x^2-2x-3}$. $\frac{0}{0}$

$$\frac{(x+4)\cancel{(x-3)}}{\cancel{(x-3)}(x+1)} = \frac{x+4}{x+1}$$

$$\text{limit} = \frac{3+4}{3+1} = \boxed{\frac{7}{4}}$$

c. $\lim_{x\to 0} \frac{x}{x+\tan x}$. $\frac{\frac{1}{x}}{\frac{1}{x}}$ $\frac{0}{0}$

$$\lim_{x\to 0} \frac{1}{1+\frac{\tan x}{x}}$$

$$\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\sin x}{x} \frac{1}{\cos x}$$

$$= 1 \cdot 1 = 1$$

$$= \frac{1}{1+1} = \frac{1}{2}$$

d. $\lim_{x\to\infty} \frac{\sqrt{9x^2+x}-3x}{1}$ $\frac{\sqrt{9x^2+x}+3x}{\sqrt{}+3x}$

$$= \lim_{x\to\infty} \frac{\cancel{9x^2}+x-\cancel{9x^2}}{\sqrt{9x^2+x}+3x} \quad \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{\frac{9x^2+x}{x^2}}+3}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{9+\frac{1}{x}}+3} = \frac{1}{\sqrt{9+0}+3}$$

$$= \frac{1}{6}$$

4. [30 points] Find the following derivatives.

a. $(x^2 \sin x)'$

$= 2x \sin x + x^2 \cos x$

b. $(x^4 - 2x^2 + 3\tan x)'$

$= 4x^3 - 4x + 3\sec^2 x$

c. $\left(\frac{7x+1}{x+3}\right)'$

$$= \frac{(7x+1)'(x+3) - (7x+1)(x+3)'}{(x+3)^2}$$

$$= \frac{7(x+3) - (7x+1)}{(x+3)^2} = \frac{2}{(x+3)^2}$$

d. $(\sqrt{x} \cdot \cos x)'$

$\frac{1}{2\sqrt{x}} \cos x - \sqrt{x} \sin x$

e. $(\cot 3x)'$

$= -\csc^2(3x) \cdot (3x)'$

$= -3 \csc^2(3x)$

f. $(\sin^3 x)'$

$= 3\sin^2 x \cdot (\sin x)'$

$= 3 \sin^2 x \cos x.$

5. [10 points] Suppose the position of a object is given, in meters, by $s = t^2 - 8t + 18$, where t is time is seconds.

a. What is the average velocity over the time interval $3 \le t \le 4$?

$9 - 24 = -15$

$$\frac{s(4) - s(3)}{4-3} = \frac{(4^2 - 8 \cdot 4 + 18) - (9 - 8 \cdot 3 + 18)}{1}$$

$$\frac{16 - 32 + 15}{1} = 31 - 32 = -1 \text{ m/s}$$

b. What is the instantaneous velocity at $t = 4$?

$s'(t) = 2t - 8$

$s'(4) = 8 - 8 = 0$

6. [10 points] Let $xy^2 - 2xy = 4$. Find y' as a function of x and y.

Apply $\frac{d}{dx}$ to both sides

$(x)'y^2 + x(y^2)' - 2y - 2xy' = 0$

$y^2 + 2xyy' - 2y - 2xy' = 0$

$(2xy - 2x)y' = -y^2 + 2y$

$$y' = \frac{2y - y^2}{2x(y-1)}$$

7. [10 points] Let $f(x)$ and $g(x)$ be differentiable functions. Prove that

$$(f(x) + g(x))' = f'(x) + g'(x).$$

$$(f(x)+g(x))' = \lim_{h\to 0} \frac{(f(x+h)+g(x+h)) - (f(x)+g(x))}{h}$$

$$= \lim_{h\to 0} \frac{(f(x+h)-f(x)) + (g(x+h)-g(x))}{h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h}$$

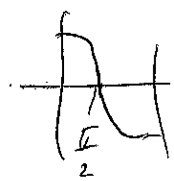
$$= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h\to 0} \frac{g(x+h)-g(x)}{h} = f'(x) + g'(x).$$

8. [10 points] For what value of x in $[0, \pi]$ does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent line? That is, when is the slope zero? Assume x is in radians.

$f'(x) = 1 + 2\cos x$. Need $f'(x) = 0$

$1 + 2\cos x = 0$

$\cos x = \frac{-1}{2}$



$x = \frac{2\pi}{3}$