Math 150 Quiz 10 Fall 2012

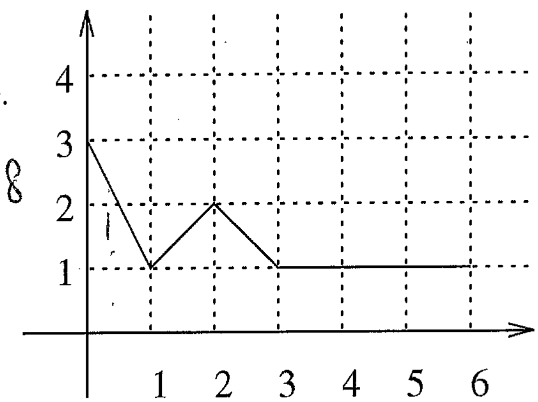
Name: ____________________ Section time: ____________________

NON-GRAPHING CALCULATORS ALLOWED

1. [20 points] The graph of a function $f(x)$ is shown in the figure below.

a. Evaluate $\int_0^6 f(x)\,dx$.

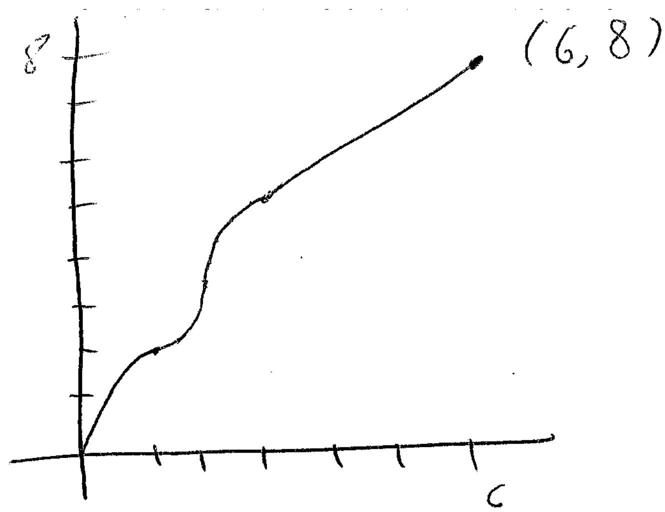
$2 + 1\frac{1}{2} + 1\frac{1}{2} + 1 + 1 + 1 = 8$



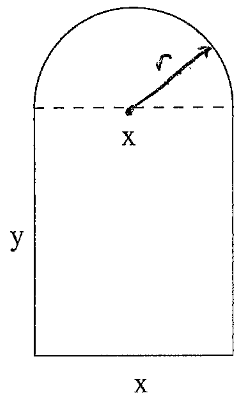
b. Determine the average value of the function in the graph on the interval [0,6].

$\frac{1}{6} \cdot 8 = \frac{4}{3}$

c. Sketch the graph of $g(x) = \int_0^x f(t)\,dt$.



2. [20 points] A Norman window is constructed by adjoining a semi-circle to the top of an ordinary rectangular window. Find the dimensions of a Norman window of maximum area if the total perimeter is 20 feet.



$r = \frac{x}{2}$

$$A = xy + \frac{\pi r^2}{2} = xy + \frac{\pi x^2}{8}$$

$$P = 20 = x + 2y + \frac{2\pi r}{2} =$$

$$20 = x + 2y + \frac{\pi x}{2}$$

$$20 = 2y + \left(\frac{\pi}{2} + 1\right)x$$

$$\frac{20 - \left(\frac{\pi}{2}+1\right)x}{2} = y$$

$$A = x\left(\frac{20 - \left(\frac{\pi}{2}+1\right)x}{2}\right) + \frac{\pi x^2}{8} = 10x - \left(\frac{\pi}{4}+\frac{1}{2}\right)x^2 + \frac{\pi}{8}x^2$$

$$= 10x - \left(\frac{\pi}{8}+\frac{1}{2}\right)x^2$$

$$\frac{dA}{dx} = 10 - 2\left(\frac{\pi}{8}+\frac{1}{2}\right)x = 10 - \left(\frac{\pi}{4}+1\right)x = 0$$

$$x = \frac{10}{\frac{\pi}{4}+1} = \frac{40}{\pi+4} \approx 5.6 \text{ ft}$$

Then $y = 10 - \left(\frac{\pi}{4}+\frac{1}{2}\right)\left(\frac{10}{\frac{\pi}{4}+1}\right)$

$$= 10\left[1 - \frac{\left(\frac{\pi}{4}+\frac{1}{2}\right)\cdot 4}{\left(\frac{\pi}{4}+1\right)\cdot 4}\right] = 10\left[1 - \frac{\pi+2}{\pi+4}\right]$$

$$= 10\left[\frac{\pi+4}{\pi+4} - \frac{\pi+2}{\pi+4}\right] = 10\left(\frac{2}{\pi+4}\right) = \frac{20}{\pi+4} \approx 2.8 \text{ ft}$$

3. [20 points] Do the integrals.

a. $\displaystyle\int_1^2 9x^2+5\,dx$ $= 3x^3+5x\Big|_1^2$

$= (3\cdot 8+5\cdot 2)-(3+5)$

$= 34-8 = 26$

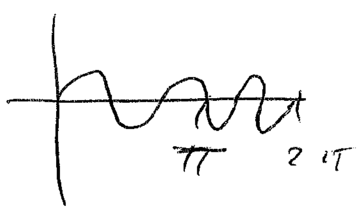
b. $\displaystyle\int \frac{x^2+\ln x}{x}\,dx$ $= \int x + \frac{\ln x}{x}\,dx$ — $\int u\,du$

$\downarrow$ $\quad$ $\hookrightarrow$ $u=\ln x$ $\quad = \frac{1}{2}u^2$

$\frac{x^2}{2}$ $\quad$ $du = \frac{1}{x}\,dx$ $\quad = \frac{1}{2}(\ln x)^2$

$\frac{x^2}{2}+\frac{(\ln x)^2}{2}+C$

c. $\displaystyle\int_0^\pi \sin(3x)\,dx$



$-\frac{1}{3}\cos(3x)\Big|_0^\pi = -\frac{1}{3}\left(\cos(3\pi)-\cos(0)\right)$

$-1 \quad - \quad 1$

$= \frac{2}{3}$

d. $\displaystyle\int (\sec^2 x)e^{\tan x}\,dx$ $\quad u=\tan x.$ $\quad du = \sec^2 x\,dx$

$= \int e^u\,du = e^u + C = e^{\tan x}+C$

e. $\displaystyle\int_{-\pi/4}^{\pi/4} \tan x\,dx$ $= 0$ odd function.

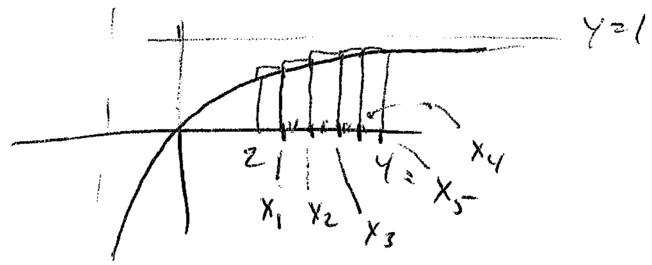
or

$\frac{\sin x}{\cos x}$ $\quad u = \cos x.$

$du = -\sin x\,dx$

$-\displaystyle\int_{\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \frac{1}{u}\,du = 0$

4. [20 points] Set up the sum for the approximate area under $\frac{x}{x+1}$ from $x = 2$ to $x = 4$ for $n = 5$. (Use the left end points, as in class.)



What is Δx?

$$\Delta x = \frac{4-2}{5} = \frac{2}{5} = 0.4$$

What are the values of x_1 through x_5?

$x_1 = 2.4$

$x_2 = 2.8$

$x_3 = 3.2$

$x_4 = 3.6$

$x_5 = 4.0$

Compute the sum to ~~five~~ 3 decimal places. (You should get ~~1.44417~~ 1.515.)

$$\left[\frac{2.4}{3.4} + \frac{2.8}{3.8} + \frac{3.2}{4.2} + \frac{3.6}{4.6} + \frac{4.0}{5.0}\right](0.4) = 1.514895166$$

Is this number less than or greater than the true area? Justify your answer. (Hint: sketch the graph and draw in the rectangles.)

5. [5 points] Let $g(x) = \int_0^{x^3} \sin t^5 \, dt$. Find $g'(x)$.

$$= F(x^3) - F(0) \quad \text{where } F'(x) = \sin x^5$$

$$g'(x) = F'(x^3) \cdot (x^3)'$$

$$= \sin(x^{15}) \cdot 3x^2$$