

Name: \_\_\_\_\_ Section time: \_\_\_\_\_

**NO CALCULATORS**

1. [5 points] What is the formal definition of the limit

$$\lim_{x \rightarrow a} f(x) = L?$$

For every  $\epsilon > 0$  there exists a  $\delta > 0$  such that if  $0 < |x - a| < \delta$  then  $|f(x) - L| < \epsilon$ .

2. [10 points] Using only the formal definition of a limit prove that

$$\lim_{x \rightarrow 2} 3x - 1 = 5.$$

Let  $\epsilon > 0$  be given. Pick  $\delta = \frac{\epsilon}{3}$ .

Suppose  $0 < |x - 2| < \delta$ . Then

$$|x - 2| < \frac{\epsilon}{3}$$

$$3|x - 2| < \epsilon$$

$$|3x - 6| < \epsilon$$

$$|(3x - 1) - 5| < \epsilon.$$

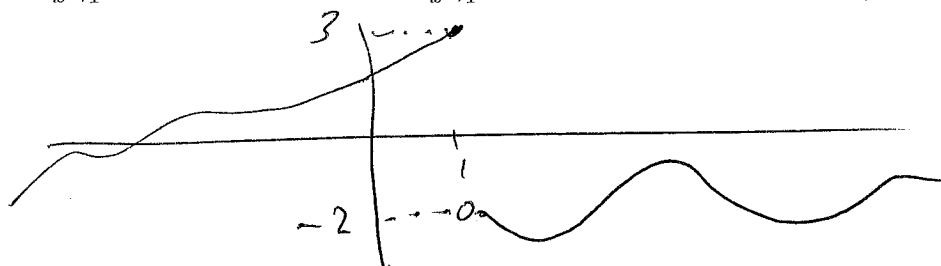
Therefore  $\lim_{x \rightarrow 2} 3x - 1 = 5$ .

3. [5 points] Sketch the graph of an example of a function  $f$  that satisfies the following conditions.

$$\lim_{x \rightarrow 1^-} f(x) = 3$$

$$\lim_{x \rightarrow 1^+} f(x) = -2$$

$$f(1) = 3$$



4. [10 points] Solve the equation below for  $x$ .

$$\ln(x+7) - \ln(x+3) = \ln 2$$

$$\ln \frac{x+7}{x+3} = \ln 2$$

$$\frac{x+7}{x+3} = 2$$

$$x+7 = 2x+6$$

$$1 = x$$

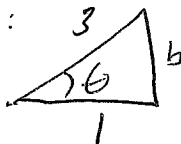
5. [5 points] What is the domain of  $\frac{\sqrt{x}}{x^2-4}$ ? Express your answer in interval notation and draw it on the real number line.

Because of the  $\sqrt{x}$ ,  $x \geq 0$ . Because  $x^2 - 4 \neq 0$  we have  $x \neq \pm 2$ . So the domain looks like



6. [5 points] Evaluate  $\tan(\sec^{-1}(3))$ .

Let  $\theta = \sec^{-1}(3)$ . Then  $\sec \theta = 3$  or  $\cos \theta = \frac{1}{3}$ .

Consider the triangle:  Then  $b = \sqrt{3^2 - 1^2} = 2\sqrt{2} = \sqrt{8}$

Thus  $\tan \theta = \frac{2\sqrt{2}}{1}$