Math 150 Key Quiz 4 A Fall 2012

Name: _____ Section time: _____

NO CALCULATORS

1. [5 points] What is the formal definition of the derivative?

The derivative of a fnction $f(x)$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \text{ when the limit exists.}$$

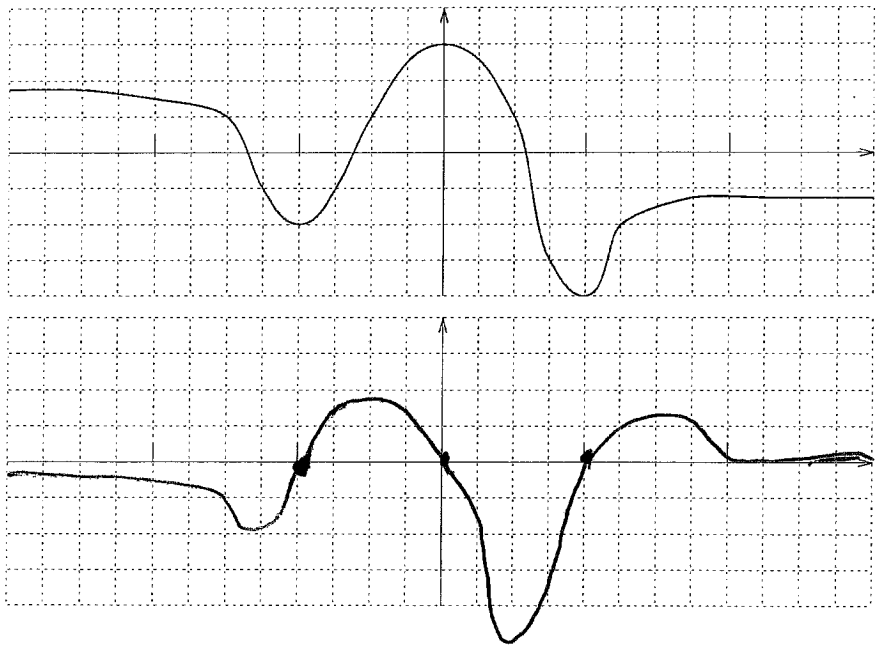
2. [10 points] Use the formal definition of the derivative to find $\left(\frac{1}{x-2}\right)'$.

$$\left(\frac{1}{x-2}\right)' = \lim_{h \to 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{h} = \lim_{h \to 0} \frac{1}{h} \frac{(x-2) - (x+h-2)}{(x+h-2)(x-2)}$$

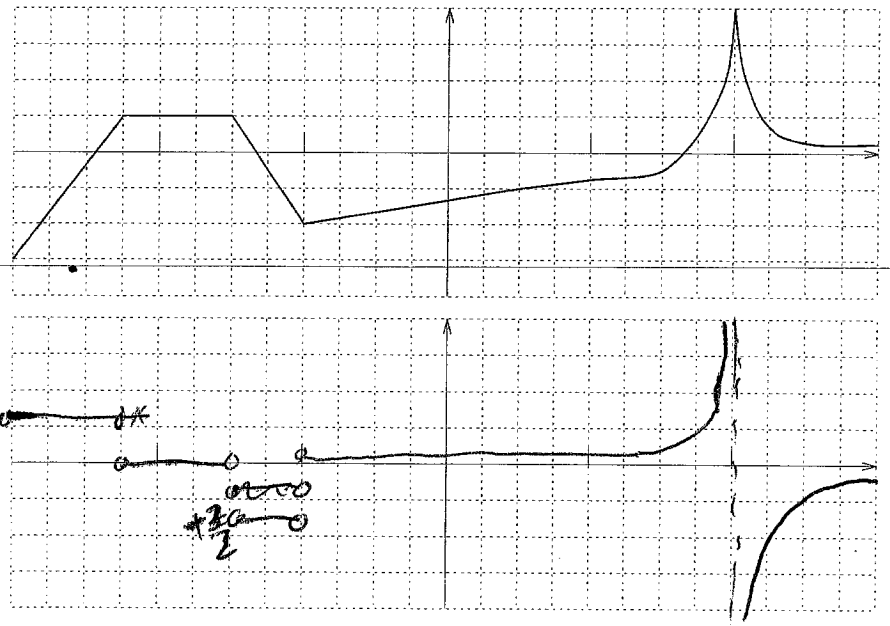
$$= \lim_{h \to 0} \frac{1}{h} \frac{-h}{(\quad)(\quad)} = \lim_{h \to 0} \frac{-1}{(x+h-2)(x-2)} = \frac{-1}{(x-2)^2}.$$

3. [10 points] Below there are two graphs of functions with an empty graph grid below each. Draw the graph of the derivative of each in the grid below its graph.

(a)



(b)



4. [15 points - 5 points each] Find the derivatives below.

a. $\left(x^3 - 6x^2 + \frac{7}{x}\right)'$

$3x^2 - 12x - \frac{7}{x^2}$

b. $(\sqrt{x} + x \sin x)'$

$\frac{1}{2\sqrt{x}} + \sin x + x \cos x$

c. $(\sin^2 x + \cos^2 x)'$ $= (1)' = 0$

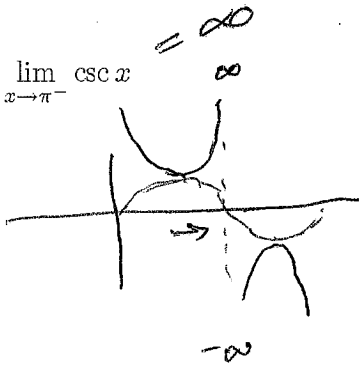
5. [20 points - 5 points each] Find the limits below. Show each step you use.

a. $\lim_{\theta \to 0} \frac{\tan 7\theta}{\theta}$ $= \lim_{\theta \to 0} \frac{7}{7} \frac{\sin 7\theta}{\theta \cos 7\theta}$

$= 7 \lim_{\theta \to 0} \frac{\sin 7\theta}{7\theta} \cdot \frac{1}{\cos 7\theta} = 7 \cdot 1 \cdot \frac{1}{1}$

$= 7$

b. $\lim_{x \to \pi^-} \csc x$ $= \infty$



$-\infty$

c. $\lim_{x \to \infty} \frac{x^2 + x + 3}{2x^2 + 5}$ $\frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim_{x \to \infty} \frac{1 + \frac{1}{x} + \frac{3}{x^2}}{2 + \frac{5}{x^2}}$

$= \frac{1 + 0 + 0}{2 + 0} = \frac{1}{2}$

d. $\lim_{x \to \infty} \frac{\sqrt{3x^2 + 2}}{5x}$ $\frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x \to \infty} \frac{\sqrt{3 + 2/x^2}}{5} = \frac{\sqrt{3 + 0}}{5}$

$= \frac{\sqrt{3}}{5}$