

Name: _____ Section time: _____

NO CALCULATORS

1. [20 points] Find the following derivatives.

a. $(\sec^2(x^3))'$

$$= 2 \sec(x^3) \cdot \sec(x^3) \tan(x^3) \cdot (x^3)'$$

$$= 6x^2 \sec^2(x^3) \tan(x^3)$$

b. $(\csc(7x^2))'$

$$= -\csc(7x^2) \cot(7x^2) \cdot (7x^2)'$$

$$= -14x \csc(7x^2) \cot(7x^2)$$

c. $(x \sin(x^2) \tan^3(2x^5))'$

$$= (x)' \sin(x^2) \tan^3(2x^5) + x(\sin(x^2))' \tan^3(2x^5) + x \sin(x^2)(\tan^3(2x^5))'$$

$$= \sin(x^2) \tan^3(2x^5) + 2x^2 \cos(x^2) \tan^3(2x^5) + x \sin(x^2) 3 \tan^2(2x^5) \sec^2(2x^5) \cdot (10x^4)$$

$$= \sin(x^2) \tan^3(2x^5) + 2x^2 \cos(x^2) \tan^3(2x^5) + 30x^5 \sin(x^2) \tan^2(2x^5) \sec^2(2x^5)$$

d. $\left[\sqrt{x^3 + \cot(\sin x)} \right]'$

$$= \frac{(x^3 + \cot(\sin x))'}{2\sqrt{x^3 + \cot(\sin x)}}$$

$$= \frac{3x^2 - \csc^2(\sin x) \cdot \cos x}{2\sqrt{x^3 + \cot(\sin x)}}$$

2. [10 points] Consider the relations
- $\sin(x^3y^2) + x \cos(xy^2) = 0$
- . Find
- $\frac{dy}{dx}$
- in terms of
- x
- and
- y
- .

$$\cos(x^3y^2) \cdot (x^3y^2)' + \cos(xy^2)x \sin(xy^2) \cdot (xy^2)' = 0' = 0$$

$$\cos(x^3y^2) \cdot (3x^2y^2 + 2x^3yy') + \cos(xy^2) + x \sin(xy^2) \cdot (y^2 + xy' y) = 0$$

$$\cos(x^3y^2)3x^2y^2 + \cos(x^3y^2)2x^3yy' + \cos(xy^2) + x \sin(xy^2)y^2 + x \sin(xy^2)xy' y = 0$$

$$[2x^3y \cos(x^3y^2) + x^2y \sin(xy^2)]y' = -3x^2y^2 \cos(x^3y^2) - \cos(xy^2) - xy^2 \sin(xy^2)$$

Finally,

$$y' = \frac{-3x^2y^2 \cos(x^3y^2) - \cos(xy^2) - xy^2 \sin(xy^2)}{2x^3y \cos(x^3y^2) + x^2y \sin(xy^2)}$$