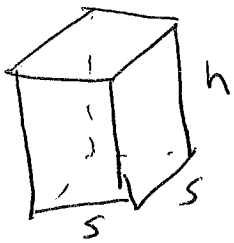
Math 150 Quiz 7 Fall 2012

Name: Solutions Section time: _____

~~NO~~ CALCULATORS OK



1. [20 points] We want to build a box container with a square base of edge length s and height h that has a volume of 100 cubic feet. Find the dimensions that will minimize the surface area? The surface area includes the bottom, top and four sides.

STEP 1: Write down the constraint equation.

$$s^2 h = 100$$

STEP 2: Write down a formula for the quanitity to be optimized.

$$A = 2s^2 + 4sh$$

STEP 3: Solve the constraint equation for one of the variables.

$$h = \frac{100}{s^2}$$

STEP 4: Substute this in the the formula from STEP 2. Simplify.

$$A = 2s^2 + 4s\left(\frac{100}{s^2}\right) = 2s^2 + \frac{400}{s}$$

STEP 5: Find the absolution minimum of the result of STEP 4.

$$\frac{dA}{ds} = 4s - \frac{400}{s^2} \overset{\text{set}}{=} 0$$

$$4s^3 = 400$$

$$s^3 = 100$$

$$s = \sqrt[3]{100} \approx 2.15443469 \text{ ft}$$

STEP 6: Record this minimum surfare area and the values of s and h

$$h = \frac{100}{(\sqrt[3]{10})^2} = \sqrt[3]{10}.$$

(so $h = s$!)

$$A = 6s^2 = 6(\sqrt[3]{10})^2 \approx 27.84955 \text{ ft}^2$$

6. Now find and simplfy $f'(x)$.

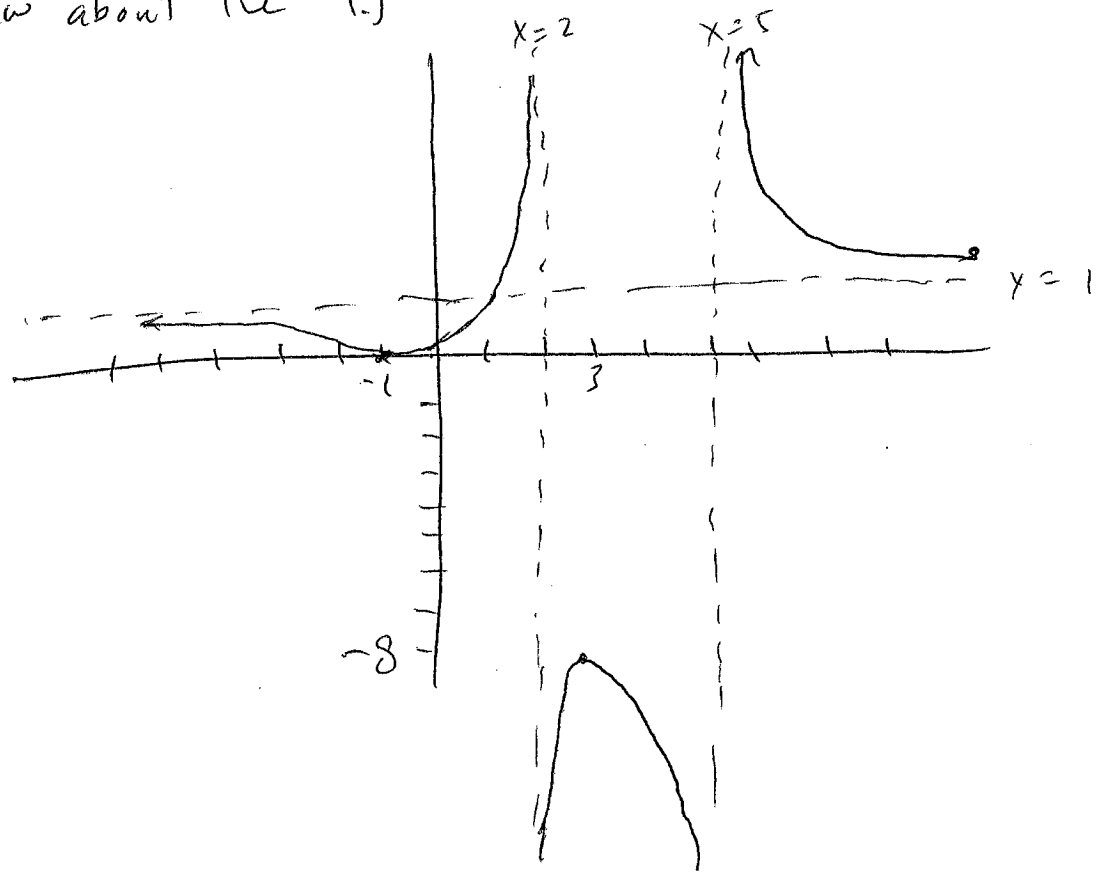
$$f'(x) = \frac{(2x+2)(x^2-7x+10) - (x^2+2x+1)(2x-7)}{(x^2-7x+10)^2}$$

$$= \frac{(2x^3-12x^2+6x+20) - (2x^3-3x^2-12x-7)}{(\quad)^2}$$

$$= \frac{-9x^2+18x+27}{(\quad)^2} = \frac{-9(x^2-2x-3)}{(\quad)^2} = -9\frac{(x-3)(x+1)}{(\quad)}$$

7. When is $f'(x) = 0$? Determine which is a local min or max. Mark these points on your graph. Refine your graph if need be.

Thus $f'(6) = 0$ for $x = 3$ and $x = -1$. (we already knew about the -1.) Note: $f(3) = -8$.



2. [20 points] Let $f(x) = \dfrac{x^2+2x+1}{x^2-7x+10}$. $= \dfrac{(x+1)^2}{(x-5)(x-2)}$

1. What are the vertical asymptotes?

$x=5 \quad x=2$

2. What is the horizontal asymptote?

$y=1$

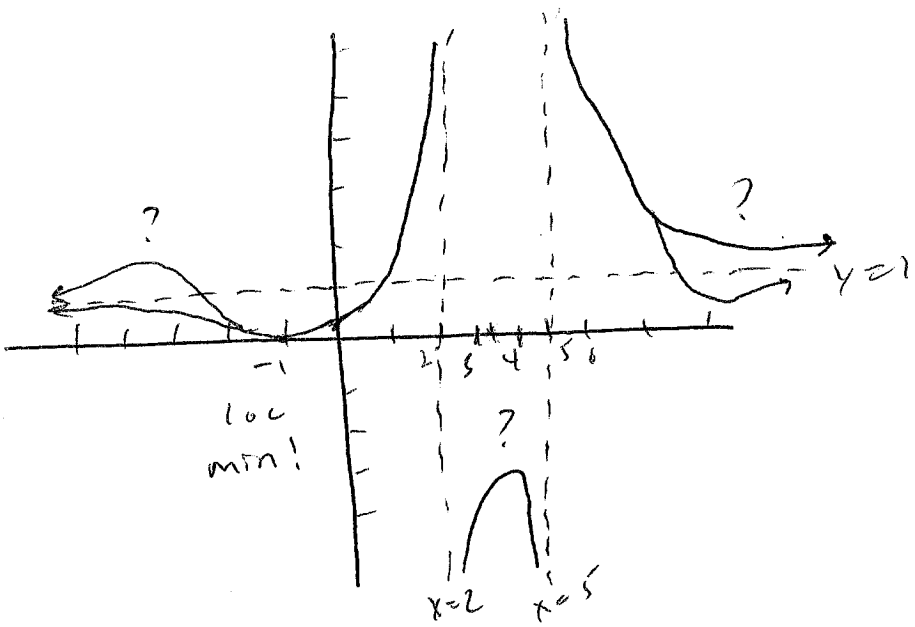
3. When is $f(x)=0$? Does the graph change sign there?

$x=-1$. NO.

4. What is $f(0)$? You can compute $f(x)$ for a couple of other values of x but you really don't need to.

$f(0) = \frac{1}{10}$

5. Sketch a rough graph of $y=f(x)$. Lable the above items.



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