Math 150 Key Test 3 Fall 2012

Name: _____ Section time: _____

SCIENTIFIC CALCULATORS ALLOWED

1. [20 points] Do the following integrals.

a. $\int \frac{\ln x}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\int u\,du = \frac{1}{2}u^2 + C$

$= \frac{1}{2}(\ln x)^2 + C$

b. $\int \sin x \sec x\,dx = \int \frac{-\sin x}{\cos x}\,dx$

$u = \cos x \quad du = -\sin x\,dx$

$= -\int \frac{1}{u}\,du = -\ln|u| + C$

$= -\ln|\cos x| + C$

$= \ln|\sec x| + C$

c. $\int \sqrt{3x+7}\,dx$

$u = 3x+7$

$du = 3\,dx$

$\frac{1}{3}\int u^{\frac{1}{2}}\,du = \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{9}(3x+7)^{3/2} + C$

d. $\int e^{3x} + xe^{x^2}\,dx = \int e^{3x}\,dx + \int x e^{x^2}\,dx$

$\frac{1}{3}e^{3x}$ $u = x^2$

$du = 2x\,dx$

$\frac{1}{2}\int e^u\,du$

$\frac{1}{3}e^{3x} + \frac{1}{2}e^{x^2} + C$

2. [5 points] Let $g(x) = \int_3^{x^3} \cot^3 t^2\,dt$. Find $\frac{dg}{dx}$.

$F(x^3) - F(3)$ where $F'(t) = \cot^3(t^2)$

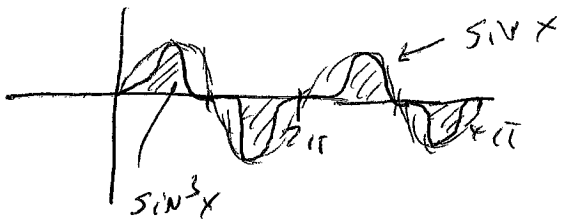
$g'(x) = (F(x^3) - F(3))' = F'(x^3)\,3x^2 - 0$

$= \cot^3(x^6)\cdot 3x^2$

3. [5 points] You and a friend are working late on a physics project that is due tomorrow. In the middle of a long calculation there is the term

$$\int_0^{4\pi} \sin^3 x \, dx.$$

You remark that it is clearly equal to zero and cross it off. But your friend says "Huh? Like, what's up with that?" Explain. You can draw a picture to help.



Since $\sin^3 x$ has the same symmetry as $\sin x$ the areas cancel.

4. [10 points] A car accelerates from 10 mph to 50 mph in 13 seconds with constant acceleration. How many feet did the car travel during those 13 seconds? Give your answer rounded to three decimal places. (Watch your units!)

1 mph = $\frac{5280}{3600}$ ft/sec

$a(t) = \frac{(50-10)\alpha}{13} = \frac{40\alpha}{13}$ α

$V(t) = \frac{40\alpha t}{13} + C$

$C = 10\alpha$, the init velocity.

$S(t) = \frac{20\alpha t^2}{13} + 10\alpha t$

$S(13) = 20\alpha \cdot 13 + 10\alpha \cdot 13 =$

$= 30 \cdot 13 \cdot \alpha = 390\alpha = 572$ ft.

572 ft. exactly

5. [20 points] A metal cylinder with radius r and height h has to hold 100 cubic feet of a fluid. We want to design this cylinder to minimize the cost C of the material used. But the top and bottom of the cylinder are required to be thicker than the side. The cost is \$5 per square foot for the metal used for the top and bottom plus \$2 per square foot for the metal used for the side.

a. Write a formula for the cost C as a function of r and h.

$$C = 5 \cdot 2 \cdot \pi r^2 + 2 \cdot 2\pi r h = 10\pi r^2 + 4\pi r h$$

b. Use the formula for the volume of this cylinder to find h as a function of r.

$$V = 100 = \pi r^2 h \Rightarrow h = \frac{100}{\pi r^2}$$

c. Plug this into the cost formula and get C in terms just of r. Simplify.

$$C = 10\pi r^2 + 4\pi r \left(\frac{100}{\pi r^2}\right) = 10\pi r^2 + \frac{400}{r}$$

d. Optimize the result of part c.

$$\frac{dC}{dr} = 20\pi r - \frac{400}{r^2} = 0$$

$$20\pi r^3 = 400$$

$$r = \left(\frac{20}{\pi}\right)^{1/3}$$

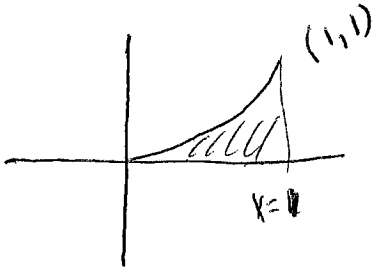
e. What is r? What is h? What is the cost C? Give your final answers in decimal form rounded to three places.

$r \approx 1.853$ ft $\quad h \approx 9.267$ ft

$C = \$323.736$

What a bargain!

6. [20=4+8+8 points] a. Draw the region bounded by $y = x^3$, the x-axis and the line $x = 1$.



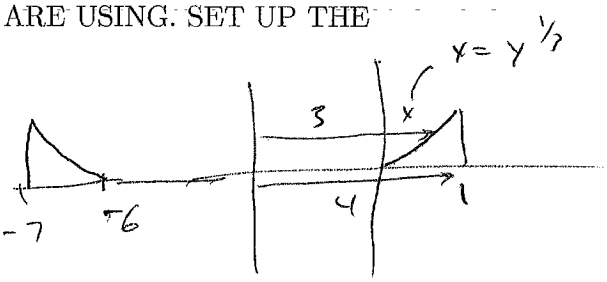
b. Find the volume formed by rotating this region about the x-axis. STATE THE METHOD YOU ARE USING. SET UP THE INTEGRAL COMPLETELY.

Disk.

$$\int_0^1 \pi (x^3)^2 \, dx = \pi \int_0^1 x^6 \, dx$$

$$= \frac{\pi}{7}$$

c. Find the volume formed by rotating this region about the line $x = -3$. STATE THE METHOD YOU ARE USING. SET UP THE INTEGRAL COMPLETELY.



Washers

$$\int_0^1 \pi 4^2 - \pi (3 + y^{1/3})^2 \, dy$$

$$= 16\pi - \pi \int_0^1 9 + 6y^{1/3} + y^{2/3} \, dy$$

$$\pi \left(9y + \frac{9}{2} y^{4/3} + \frac{3}{5} y^{5/3} \right) \Big|_0^1$$

$$\left(16 - \left(9 + \frac{9}{2} + \frac{3}{5}\right)\right)\pi$$

$$\frac{27}{2}$$

$$= 1.9\pi$$

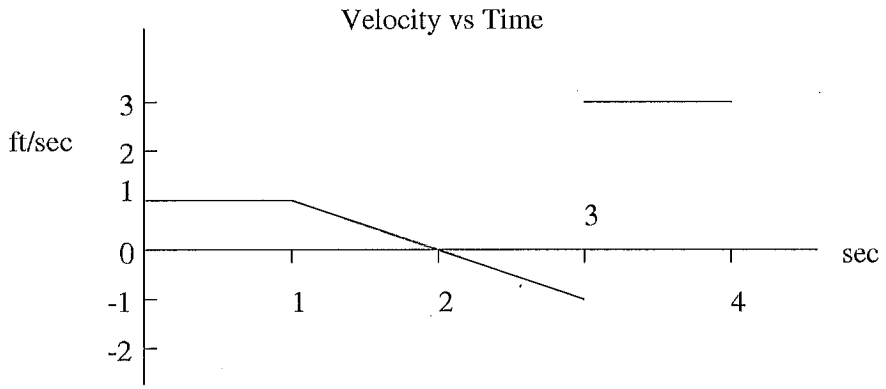
Shells

$$\int_0^1 2\pi x (3 + x^3) \, dx$$

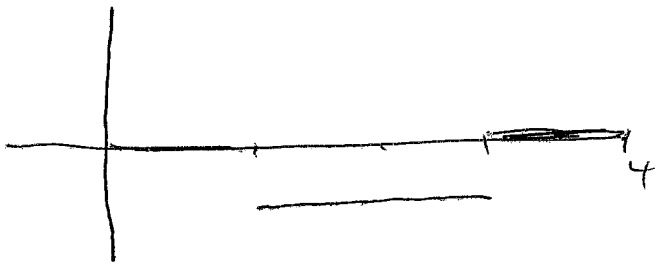
$$= 2\pi \int_0^1 3x^3 + x^4 \, dx$$

$$= 2\pi \left(\frac{3}{4} + \frac{1}{5} \right) = 1.9\pi$$

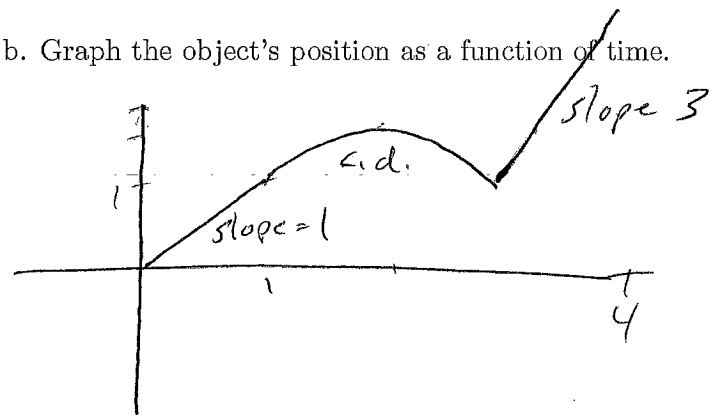
7. [20=3+9+4+4 points] An object's velocity as a function of time is given by the graph below.



a. Graph the object's acceleration as a function of time.



b. Graph the object's position as a function of time.



c. Where is the object relative to its starting point after 4 seconds?

4 $(1 + \frac{1}{2} - \frac{1}{2} + 3)$

d. What is the total number of feet traveled by the object during the 4 seconds?

5 $(1 + \frac{1}{2} + \frac{1}{2} + 3)$