Math 150 Test 1 Fall 2012

Name: ____________________ Section time: ____________________

NO CALCULATORS

1. [30 points] Find the limits. You must show your work.

a. $\lim_{x\to 0} \frac{\sin 2x}{\sin 4x} = \lim_{x\to 0} \frac{\sin 2x}{x} \cdot \frac{x}{\sin 4x}$

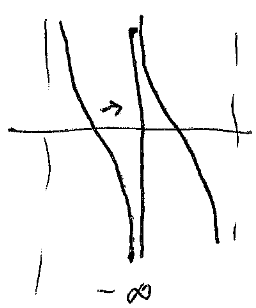
$= \frac{2}{4} \lim_{x\to 0} \frac{\sin 2x}{2x} \cdot \frac{4x}{\sin 4x}$

$= \frac{1}{2} \cdot 1 \cdot \frac{1}{1} = \frac{1}{2}$

b. $\lim_{x\to 3} \frac{x^2+x-12}{x^2-2x-3}$ $\frac{0}{0}$ $= \lim_{x\to 3} \frac{(x+4)(x-3)}{(x+1)(x-3)}$

$= \lim_{x\to 3} \frac{x+4}{x+1} = \frac{3+4}{3+1} = \frac{7}{4}$

c. $\lim_{x\to 0-} \cot x = -\infty$



d. $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2} \cdot \frac{1+\cos\theta}{1+\cos\theta} =$

$\lim_{\theta\to 0} \frac{1-\cos^2\theta}{\theta^2} \cdot \frac{1}{1+\cos\theta}$

$= \lim_{\theta\to 0} \frac{\sin^2\theta}{\theta^2} \cdot \frac{1}{1+\cos\theta}$

$= 1^2 \cdot \frac{1}{1+1} = \frac{1}{2}$

e. $\lim_{t\to 16} \frac{4-\sqrt{t}}{t-16} \cdot \frac{4+\sqrt{t}}{4+\sqrt{t}}$

$= \lim_{t\to 16} \frac{16-t}{t-16} \cdot \frac{1}{4+\sqrt{t}}$

$= \lim_{t\to 16} \frac{-1}{4+\sqrt{t}} = \frac{-1}{4+4} = \frac{-1}{8}$

f. $\lim_{x\to 0} \sin(x+\tan x)$

$= \sin(0+\tan(0))$

$= \sin(0) = 0$

2. [30 points] Find the following derivatives using differentiation rules we have covered.

a. $(\pi^3 + \tan 3x)'$

$0 + \sec^2(3x) \cdot (3x)'$

$= 3\sec^2(3x)$

b. $(x \cos x + \tan x^3)'$

$(x)'\cos x + x(\cos x)' + \sec^2(x^3)(x^3)'$

$\cos x - x \sin x + 3x^2 \sec(x^3)$

c. $\left(\dfrac{\sin x^2}{x^3+2}\right)'$

$$= \frac{(\sin x^2)'(x^3+2) - \sin(x^2)(x^3+2)'}{(x^3+2)^2}$$

$$= \frac{2x\cos x(x^3+2) - 3x^2 \sin(x^2)}{(x^3+2)^2}$$

d. $(x^2 + 3 + \sin 2x)''''$

$2x+0$ $2\cos 2x$

2 $-4\sin(2x)$

0 $-8\cos(2x)$

0 $\boxed{16\sin(2x)}$

e. $((x^2+1)\sec x)'$

$(x^2+1)'\sec x + (x^2+1)(\sec x)'$

$2x\sec x + (x^2+1)\sec x \tan x$

f. $\left(\dfrac{1}{x^3}\right)'' = \left(\dfrac{-3}{x^4}\right)' = \dfrac{12}{x^5}$

or

$(x^{-3})'' = (-3x^{-4})' = 12x^{-5}$

3. [10 points] Find the derivative of $f(x) = \sqrt{x+1}$ using only the formal definition of the derivative and properties of limits.

$$\left(\sqrt{x+1}\right)' = \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{} + \sqrt{}}{\sqrt{} + \sqrt{}} =$$

$$\lim_{h \to 0} \frac{(x+h+1) - (x+1)}{h\left(\sqrt{} + \sqrt{}\right)} = \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+h+1} + \sqrt{x+1}\right)} =$$

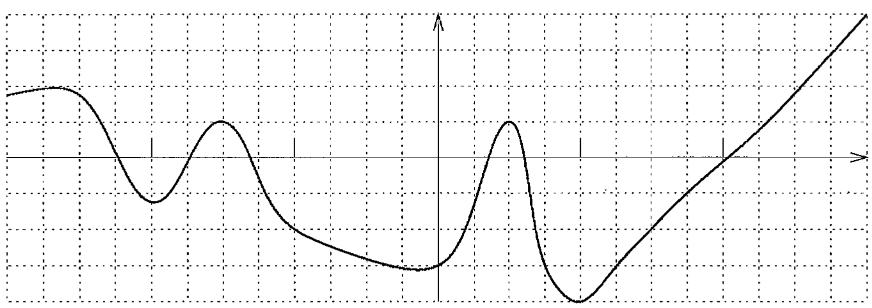
$$\lim_{h \to 0} \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} = \frac{1}{\sqrt{x+0+1} + \sqrt{x+1}} = \frac{1}{2\sqrt{x+1}}$$

4. [5 points] Use the Intermediate Value Theorem to show that $f(x) = x^3 + x + 1$ has at least one real zero in the interval $[-1, 0]$.

$f(-1) = -1$ $\quad -1 < 0 < 1$. By the IVT $\exists\, c \in (-1, 0)$

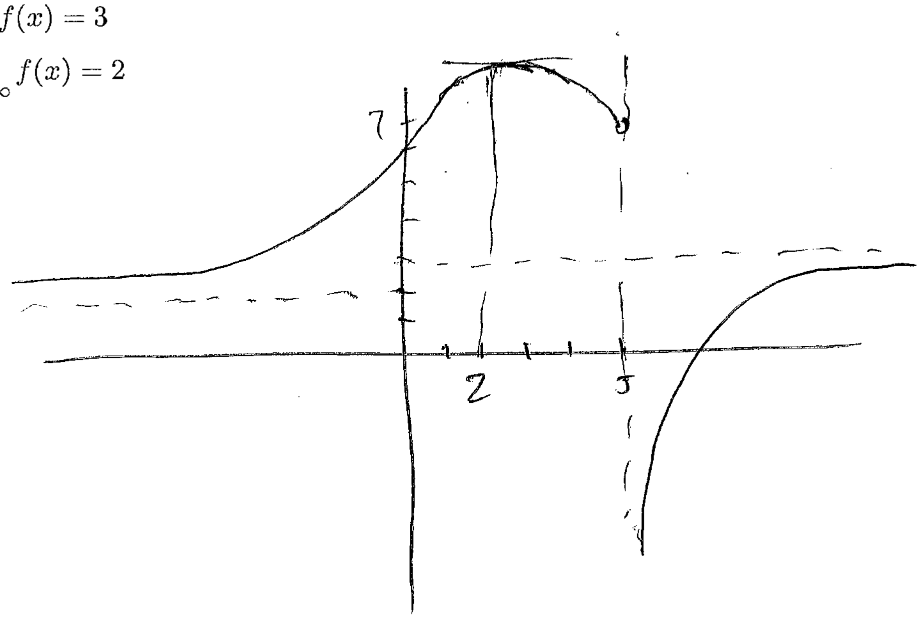
$f(0) = 1$ $\quad$ s.t. $f(c) = 0$.

5. [5 points] Let $y = f(x)$ be determined by the graph below. In the empty grid below it sketch a graph of the derivative.



6. [10 points] Sketch a graph of a single function $y = f(x)$ that has the following properties.

   (a) $f'(2) = 0$
   (b) $\lim_{x \to 5^-} f(x) = 7$
   (c) $\lim_{x \to 5^+} f(x) = -\infty$
   (d) $\lim_{x \to \infty} f(x) = 3$
   (e) $\lim_{x \to -\infty} f(x) = 2$



7. [10 points] Consider the relation $x^2 + 2xy^3 + y = 4$. Find the tangent line passing through $(1, 1)$. Express your final answer in slope-intercept form.

$$2x + 2y^3 + 6xy^2y' + y' = 0$$

$$(6xy^2 + 1)y' = -2x - 2y^3$$

$$y' = \frac{-2x - 2y^3}{6xy^2 + 1} = \frac{-4}{7}$$

$$y - 1 = -\frac{4}{7}(x - 1)$$

$$y = -\frac{4}{7}x + \underbrace{\frac{4}{7} + 1}\ \frac{7}{7}$$

$$y = \frac{-4}{7}x + \frac{11}{7}$$