Math 150 Test 1 Fall 2012

Name: ______________________ Section time: ______________________

NO CALCULATORS

1. [30 points] Find the limits. You must show your work.

a. $\lim\limits_{x\to 0} \dfrac{\sin 2x}{\sin 4x} = \lim\limits_{x\to 0} \dfrac{\sin 2x}{x} \dfrac{x}{\sin 4x}$

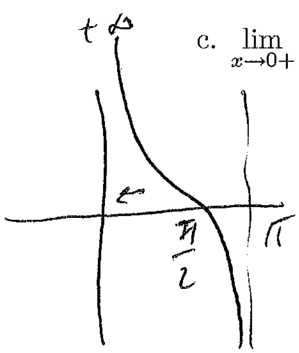
$= \dfrac{2}{4} \lim\limits_{x\to 0} \dfrac{\sin 2x}{2x} \dfrac{4x}{\sin 4x}$

$= \dfrac{1}{2} \cdot 1 \cdot 1 = \dfrac{1}{2}$

b. $\lim\limits_{x\to 3} \dfrac{x^2+x-12}{x^2-2x-3} = \lim\limits_{x\to 3} \dfrac{(x+4)(x-3)}{(x+1)(x-3)}$

$= \lim\limits_{x\to 3} \dfrac{x+4}{x+1} = \dfrac{3+4}{3+1} = \dfrac{7}{4}$

c. $\lim\limits_{x\to 0+} \cot x = \infty$



d. $\lim\limits_{\theta\to 0} \dfrac{1-\cos\theta}{\theta^2} \quad \dfrac{1+\cos\theta}{1+\cos\theta}$

$\lim\limits_{\theta\to 0} \dfrac{1-\cos^2\theta}{\theta^2(1+\cos\theta)} = \lim\limits_{\theta\to 0} \dfrac{\sin^2\theta}{\theta^2} \dfrac{1}{(1+\cos\theta)}$

$= 1 \cdot \dfrac{1}{1+1} = \dfrac{1}{2}$

e. $\lim\limits_{t\to 16} \dfrac{4-\sqrt{t}}{t-16} \quad \dfrac{4+\sqrt{t}}{4+\sqrt{t}}$

$= \lim \dfrac{16-t}{t-16} \dfrac{1}{4+\sqrt{t}}$

$= -\lim\limits_{t\to 16} \dfrac{1}{4+\sqrt{t}} = -\dfrac{1}{4+4}$

$= \dfrac{-1}{8}$

f. $\lim\limits_{x\to 0} \cos(x+\sin x)$

$= \cos(0+\sin(0))$

$= \cos(0+0)$

$= \cos(0) = 1.$

2. [30 points] Find the following derivatives using differentiation rules we have covered.

a. $(\pi^3 + \tan 2x)'$

$= 0 + \sec^2(2x) \cdot (2x)'$

$= 2\sec^2(2x)$

b. $((x^2+1)\sec x)'$

$= (x^2+1)' \sec x + (x^2+1)(\sec x)'$

$= 2x \sec x + (x^2+1) \sec x \tan x$

c. $\left(\dfrac{\sin x^2}{x^3+2}\right)' =$

$$\frac{(\sin(x^2))'(x^3+2) - \sin(x^2)(x^3+2)'}{(x^3+2)^2}$$

$$= \frac{2x\cos(x^2)(x^3+2) - 3x^2\sin(x^2)}{(x^3+2)^2}$$

d. $(\underbrace{x^2+3}+\sin 2x)''''$

$2x \quad 2\cos 2x$

$2 \quad -4\sin 2x$

$0 \quad -8\cos 2x$

$+16\sin 2x$

e. $(x\cos x + \tan x^3)'$

$(x)'\cos x + x(\cos x)' + \sec^2(x^3)(x^3)'$

$\cos x - x\sin x + 3x^2\sec^2(x^3).$

f. $\left(\dfrac{1}{x^3}\right)'' = \left(\dfrac{-3}{x^4}\right)' =$

$\dfrac{+12}{x^5}$

or $(x^{-3})'' = (-3x^{-4})'$

$= 12x^{-5}.$

3. [5 points] Use the Intermediate Value Theorem to show that $f(x) = x^3 + x + 1$ has at least one real zero in the interval $[-1, 0]$.

$f(-1) = -1$ By the IVT $\exists\, c \in (-1, 0)$

$f(0) = 1$ s.t. $f(c) = 0$.

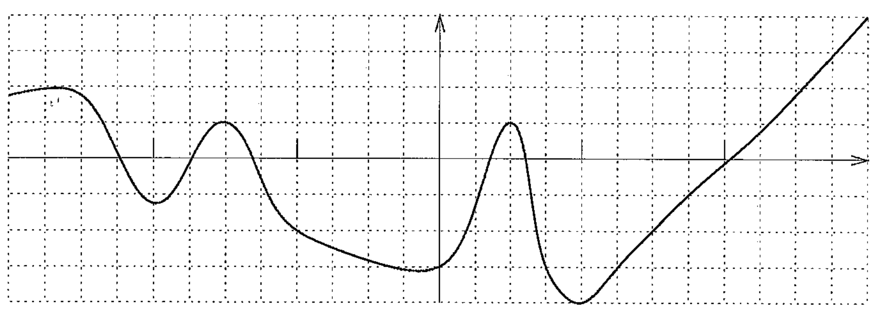
4. [10 points] Find the derivative of $f(x) = \sqrt{x+1}$ using only the formal definition of the derivative and properties of limits.

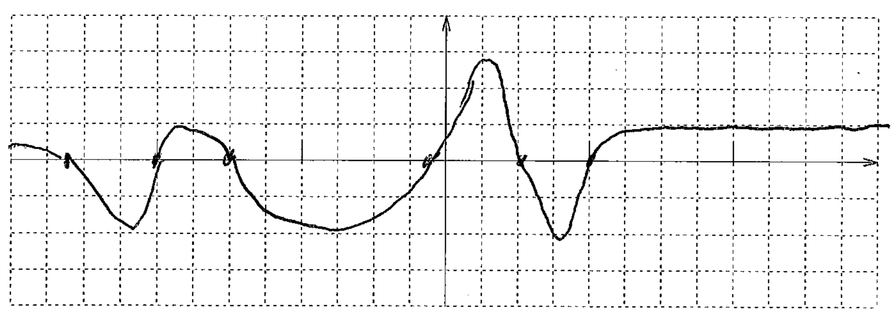
$$(\sqrt{x+1})' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{\quad} + \sqrt{\quad}}{\sqrt{\quad} + \sqrt{\quad}}$$

$$= \lim_{h \to 0} \frac{1}{h} \frac{((x+h+1) - (x+1))}{\sqrt{x+h+1} + \sqrt{x+1}} = \lim_{h \to 0} \frac{1}{\cancel{h}} \frac{\cancel{h}}{\sqrt{x+h+1} + \sqrt{x+1}} = \frac{1}{\sqrt{x+0+1} + \sqrt{x+1}}$$

$$= \frac{1}{2\sqrt{x+1}}$$

5. [5 points] Let $y = f(x)$ be determined by the graph below. In the empty grid below it sketch a graph of the derivative.





6. [10 points] Sketch a graph of a single function $y = f(x)$ that has the following properties.

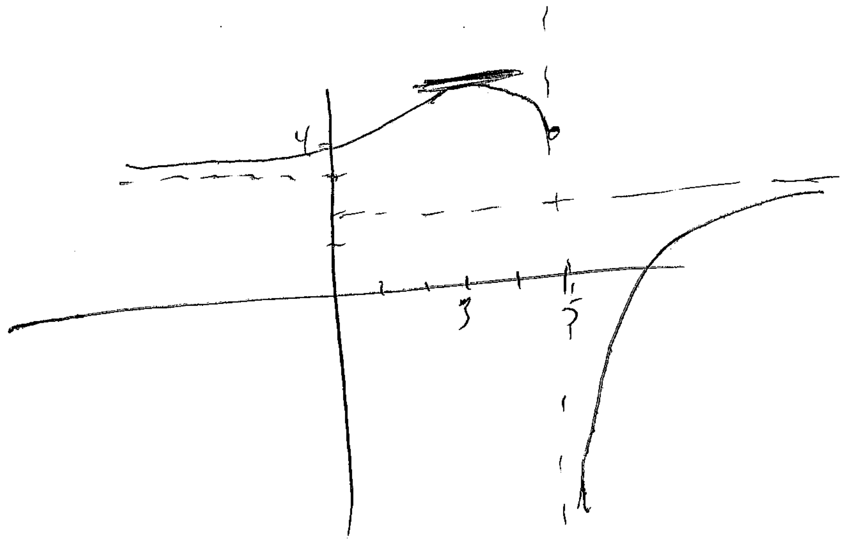
(a) $f'(3) = 0$

(b) $\lim_{x\to 5^-} f(x) = 4$

(c) $\lim_{x\to 5^+} f(x) = -\infty$

(d) $\lim_{x\to \infty} f(x) = 2$

(e) $\lim_{x\to -\infty} f(x) = 3$



7. [10 points] Consider the relation $x^2 + 2xy^3 + y = 4$. Find the tangent line passing through $(1, 1)$. Express your final answer in slope-intercept form.

$$2x + 2y^3 + 6xy^2y' + y' = 0$$

$$(6xy^2+1)y' = -2x - 2y^3$$

$$y' = \frac{-2x-2y^3}{6xy^2+1} = \frac{-4}{7}$$

$$y - 1 = -\frac{4}{7}(x-1)$$

$$y = -\frac{4}{7}x + \frac{11}{7}$$