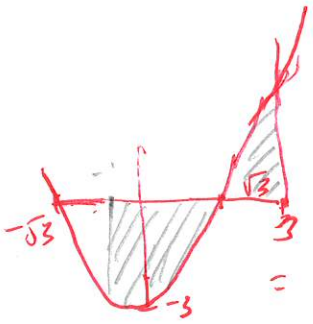
Math 150 Quiz 10 Fall 2024

Name: _____



1. [5 points each] Find the definite integrals.

a. $\int_{-1}^{3} x^2 - 3\,dx$

$= \frac{x^3}{3} - 3x \Big|_{-1}^{3}$

$= (9-9) - \left(\frac{-1}{3}+3\right) = \frac{-8}{3}$

or $-2\frac{2}{3}$

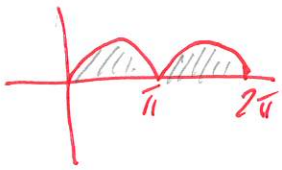
b. $\int_1^4 \frac{4+6t}{\sqrt{t}}\,dt = \int_1^4 4t^{-\frac{1}{2}} + 6t^{\frac{1}{2}}\,dt$

$= \frac{4t^{\frac{1}{2}}}{\frac{1}{2}} + \frac{6t^{\frac{3}{2}}}{\frac{3}{2}} \Big|_1^4$

$= 8t^{\frac{1}{2}} + 4t^{3/2} \Big|_1^4 = (8\cdot 2 + 4\cdot 2^3) - (8+4)$

$8\cdot 6$

$48 - 12 = 36$



c. $\int_0^{2\pi} |\sin x|\,dx$

$= \int_0^{\pi} \sin x\,dx + \int_{\pi}^{2\pi} -\sin x\,dx$

But, these have to be =.

$\int_0^{\pi} \sin x\,dx = -\cos x \Big|_0^{\pi} = (-(-1) - (-1)) = 2$

Thus $\int_0^{2\pi} |\sin x|\,dx = 4$

d. $\int_{-1}^{1} 7x^5 + \sin(x^3)\,dx = 0$

↑ odd function ↑ odd function

2. [5 points each] Find the indefinite integrals. Don't forget the $+C$.

a. $\int \frac{3}{1+4x^2}\,dx$

$= 3\int \frac{1}{1+(2x)^2}\,dx$

~~$\neq$~~ $3\arctan(2x)$?

Not quite. Need $\div 2$.

$= \frac{3}{2}\arctan(2x) + C$

b. $\int \frac{1}{\cos^2 x}\,dx = \int \sec^2 x\,dx = \tan x + C$

3. [5 points] Let $g(x) = \int_0^{\ln x} \tanh(t)\, dt$. Find $g'(x)$.

Let $F(x)$ be an antiderivative of $\tanh(x)$. Thus $F'(x) = \tanh(x)$

Then $g(x) = F(\ln x) - F(0)$.

Thus, $g'(x) = [F(\ln x) - F(0)]' = \tanh(\ln x) \cdot (\ln x)' - 0$

($F(0)$ is a constant)

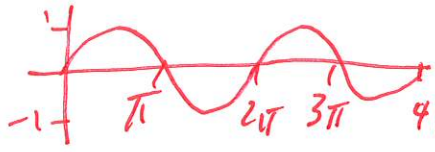
$= \dfrac{\tanh(\ln x)}{x}$.

4. [5 points] You and a friend are working late on a physics project that is due tomorrow. In the middle of a long calculation there is the term

$$\int_0^{4\pi} \sin^3 x\, dx.$$

You remark that it is clearly equal to zero and cross it off. But your friend says "Huh? Like, what's up with that?" Explain. You can draw a picture to help.

Graph of $y = \sin x$ over $[0, 4\pi]$ is



pos. and neg. areas cancel.

Graph of $y = (\sin x)^3$ is (roughly)

pos. and neg. areas still cancel.

Thus, $\int_0^{4\pi} \sin^3 x\, dx = 0$.