Name: ____________________________

Math 150 Quiz 3 Fall 2024

SCIENTIFIC CALCULATORS ALLOWED

(1) [5 points each] Find the following limits. Show your work.

a. $\lim_{x\to 3} \frac{x-3}{x^3-27}$ "$\frac{0}{0}$"

$$\begin{array}{r} x^2+3x+9 \\ x-3 \overline{) x^3 \qquad -27} \\ -(x^3-3x^2) \\ \hline 3x^2 \quad -27 \\ -(3x^2-9x) \\ \hline 9x-27 \\ 9x-27 \\ \hline 0 \end{array}$$

Thus, $x^3-27 = (x-3)(x^2+3x+9)$

$= \lim_{x\to 3} \frac{1}{x^2+3x+9}$

$= \frac{1}{9+9+9}$

$= \frac{1}{27}$

b. $\lim_{x\to\infty} \frac{3x+2}{7x+1} \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{3+\frac{2}{x}}{7+\frac{1}{x}} = \frac{3+0}{7+0} = \frac{3}{7}$

c. $\lim_{x\to 0} \frac{\sin 6x}{\sin 4x} = \lim_{x\to 0} \frac{6}{4} \cdot \frac{\sin 6x}{6x} \cdot \frac{4x}{\sin 4x}$

$= \frac{6}{4} \lim_{x\to 0} \frac{\sin 6x}{6x} \cdot \lim \frac{4x}{\sin 4x}$

$= \frac{3}{2} \cdot 1 \cdot 1 = \frac{3}{2}$

d. $\lim_{x\to\infty} \frac{\sqrt{9x^2+x} - 3x}{1} \cdot \frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x}$

$= \lim_{x\to\infty} \frac{\cancel{9x^2}+x-\cancel{9x^2}}{\sqrt{9x^2+x}+3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{1}{\sqrt{9+\frac{1}{x}}+3}$

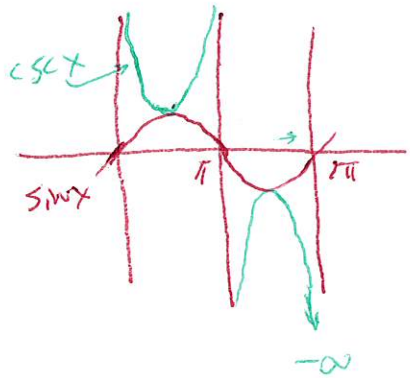
$= \frac{1}{\sqrt{9+0}+3} = \frac{1}{6}$

e. $\lim_{x\to 2\pi^-} x \csc x$

$= \lim_{x\to 2\pi^-} x \cdot \lim_{x\to 2\pi^-} \csc x$

$= 2\pi(-\infty) = -\infty$

by rules for infinite limits.



f. $\lim_{x\to\infty} \frac{2-x}{(x-1)^2} = \lim_{x\to\infty} \frac{2-x}{x^2-2x+1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\frac{2}{x^2}-\frac{1}{x}}{1-\frac{2}{x}+\frac{1}{x^2}}$

$= \frac{0-0}{1-0+0} = 0$

or, limit $= 0$ since the degree of the denominator is greater than the degree of the numerator.

This is Example 8 in Section 1.5

(2) [5 points] Use the Intermediate Value Theorem to show there is a root of the equation

$$\text{Let } f(x) = 4x^3 - 6x^2 + 3x - 2 = 0$$

between 1 and 2.

$f(1) = 4 - 6 + 3 - 2 = -2 + 1 = -1 < 0.$

$f(2) = 4 \cdot 8 - 6 \cdot 4 + 3 \cdot 2 - 2$

$= 32 - 24 + 6 - 2$

$= 8 + 4 = 12 > 0.$

Since polynomials are continuous and $f(1) < 0$ while $f(2) > 0$, The I.V.T. tells us there exists a value $c \in (1,2)$ such that $f(c) = 0$.

This is #33 in Section 1.5.

(3) [5 points] Find the value of c such that the function below is continuous on all of $\mathbb{R} = (-\infty, \infty)$. Show how you solve for c.

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

We need the two branches of f to match at $x = 2$.

Thus, $4c + 4 = 8 - 2c$

$6c = 4$

$c = \frac{4}{6} = \frac{2}{3}.$

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$= \lim\limits_{x\to 0} \frac{6}{4} \frac{\sin 6x}{6x} \frac{4x}{\sin 4x}$

$= \frac{6}{4} \lim\limits_{x\to 0} \frac{\sin 6x}{6x} \cdot \lim\limits_{x\to 0} \frac{4x}{\sin 6x}$

$= \frac{6}{4} \cdot 1 \cdot 1 = \frac{3}{2}$

b. $\lim\limits_{x\to\infty} \frac{5x+2}{3x+1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim\limits_{x\to\infty} \frac{5+\frac{2}{x}}{3+\frac{1}{x}}$

$= \frac{5+0}{3+0} = \frac{5}{3}$

c. $\lim\limits_{x\to 3} \frac{x-3}{x^3-27}$ "$\frac{0}{0}$"

$$
\begin{array}{r}
x^2+3x+9 \\
x-3 \overline{\big)\, x^3 \qquad -27} \\
-(x^3-3x^2 \qquad) \\
\hline
3x^2 \qquad -27 \\
-(3x^2-9x \qquad) \\
\hline
9x-27 \\
-(9x-27) \\
\hline
0
\end{array}
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Thus, $x^3-27 = (x-3)(x^2+3x+9)$.

$\lim\limits_{x\to 3} \frac{x-3}{(x-3)(x^2+3x+9)}$

$= \lim\limits_{x\to 3} \frac{1}{x^2+3x+9}$

$= \frac{1}{3^2+3\cdot 3+9} = \frac{1}{9+9+9} = \frac{1}{27}$

d. $\lim_{x\to\infty} \frac{\sqrt{9x^2+x}-3x}{1} \cdot \frac{\sqrt{9x^2+x}+3x}{\sqrt{9x^2+x}+3x}$

$= \lim_{x\to\infty} \frac{\cancel{9x^2}+x-\cancel{9x^2}}{\sqrt{9x^2+x}+3x} \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{1}{\sqrt{9+\frac{1}{x}}+3} = \frac{1}{\sqrt{9+0}+3} = \frac{1}{6}$

e. $\lim_{x\to\infty} \frac{2-x}{(x-1)^2} = \lim_{x\to\infty} \frac{-x+2}{x^2-2x+1} = 0$

Since degree of denominator is greater than degree of the numerator.

Or, $= \lim_{x\to\infty} \frac{2-x}{x^2-2x+1} \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim_{x\to\infty} \frac{\frac{2}{x^2}-\frac{1}{x}}{1-\frac{2}{x}+\frac{1}{x^2}} = \frac{0-0}{1-0+0}$

$= 0.$

f. $\lim_{x\to 2\pi^-} x\csc x = \lim_{x\to 2\pi^-} x \lim_{x\to 2\pi^-} \csc x$

$= 2\pi(-\infty) = -\infty$

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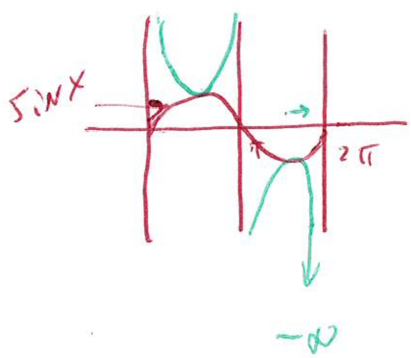

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