Name: ______________________________

Math 150 Quiz 4 Fall 2024

SCIENTIFIC CALCULATORS ALLOWED

(1) [5 points each] Use the definition of a derivative and the properties of limits to find the derivatives of the two functions below.

a. $f(x) = \frac{1}{x+1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cancel{(x+1)} - (\cancel{x}+h+\cancel{1})}{(x+h+1)(x+1)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-\cancel{h}}{(x+h+1)(x+1)}}{\cancel{h}} = \lim_{h \to 0} \frac{-1}{(x+h+1)(x+1)}$$

$$= \frac{-1}{(x+0+1)(x+1)} = \frac{-1}{(x+1)^2}$$

b. $f(x) = \sqrt{x}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{x+h - x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

(2) [2 points each] Find the derivative with respect to x of each function below.

a. $f(x) = 5x^6$

$f'(x) = 30x^5$

b. $g(x) = \frac{x+3}{x}$ (Simplify first.)

$g(x) = 1 + \frac{3}{x}$

$g'(x) = \frac{-3}{x^2}$

c. $h(x) = \pi^3$ (Trick question.)

$h'(x) = 0$

d. $k(x) = x + 2\cos x$

$k'(x) = 1 - 2\sin x$

e. $p(x) = 5 + 2x^3 + \sin x$

$p'(x) = 6x^2 + \cos x$

(3) [10 points] You project a ball straight up from the ground with an initial velocity of 50 ft/sec. When will it hit the ground? How high will it go? (Neglect air resistance. Assume $g = 32$ ft/sec/sec is the downward acceleration due to gravity.)

$a(t) = -32$

$v(t) = -32t + \underline{50}$

$s(t) = -16t^2 + 50t + \underline{0}$

Set $s(t) = 0$

$-16t^2 + 50t = 0$

$t(-16t + 50) = 0$

$t = 0$ starting time

$t = \frac{50}{16} = \frac{25}{4} = 6.25$ seconds it will hit the ground

Set $v(t) = 0$

$t = \frac{-50}{-32} = \frac{25}{16}$

$s\left(\frac{25}{16}\right) = -16\left(\frac{25}{16}\right)^2 + 50\left(\frac{25}{16}\right)$

$= 39.0625$ ft.

(39 is close enough.)

(4) [5 points each] Find the limits. Show your work.

a. $\lim_{x \to 0} \frac{\cos 2x}{\cos 3x}$

$= \frac{\cos(2 \cdot 0)}{\cos(3 \cdot 0)}$

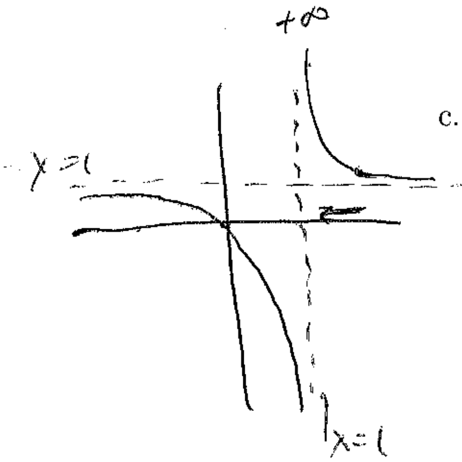
$= \frac{\cos(0)}{\cos(0)} = \frac{1}{1} = 1.$

b. $\lim_{x \to \infty} \frac{\sqrt{x^2+2x+3} - x}{1} \cdot \frac{\sqrt{x^2+2x+3} + x}{\sqrt{x^2+2x+3} + x}$

$= \lim_{x \to \infty} \frac{x^2 + 2x + 3 - x^2}{\sqrt{x^2+x+3} + x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x \to \infty} \frac{2 + \frac{3}{x}}{\sqrt{1 + \frac{1}{x} + \frac{3}{x^2}} + 1}$

$= \frac{2 + 0}{\sqrt{1+0+0} + 1} = \frac{2}{2} = 1$



c. $\lim_{x \to 1^+} \frac{x}{x-1} = \infty$

For $x > 1$

$\frac{x}{x-1}$ is positive.

~~As~~ $x \to 1$ from the right side limit is ∞.

c. $\lim_{x \to \infty} \frac{2x^2+2}{5x^2+3} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$

$= \lim_{x \to \infty} \frac{2 + \frac{2}{x^2}}{5 + \frac{3}{x^2}} = \frac{2+0}{5+0} = \frac{2}{5}$

Or, cite thm about limit of ratios of polynomials.