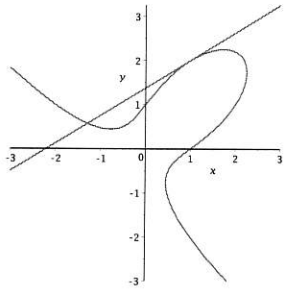
Name: _____

Math 150 Quiz 5 Fall 2024

ONLY SCIENTIFIC CALCULATORS ALLOWED

(1) [5 points each] Consider the relation $x^3 + y^3 = 4xy + 1$. Find the equation for the line tangent to its graph at the point (1,2). Express your answer in slope-intercept form. (This is similar to Example 2 in Section 2.6.)



Apply $\frac{d}{dx}$ to both sides.

$3x^2 + 3y^2y' = 4y + 4xy'$

Let $x=1$, $y=2$.

$3 + 12y' = 8 + 4y'$

$8y' = 5$

$y' = 5/8$

Tangent line is $y - 2 = \frac{5}{8}(x-1)$,

or $y = \frac{5}{8}x + \frac{11}{8}$

(2) [10 points each] Given $\tan(x/y) = x + y$, find dy/dx by implicit differentiation. (This is Exercise 11 in Section 2.6)

Apply $\frac{d}{dx}$ to both sides.

$\sec^2\left(\frac{x}{y}\right)\left(\frac{x}{y}\right)' = 1 + y'$

$\sec^2\left(\frac{x}{y}\right)\left(\frac{x'y - xy'}{y^2}\right) = 1 + y'$

$\sec^2\left(\frac{x}{y}\right)(y - xy') = y^2 + y^2y'$

$y\sec^2\left(\frac{x}{y}\right) - x\sec^2\left(\frac{x}{y}\right)y' = y^2 + y^2y'$

$\left(-x\sec^2\left(\frac{x}{y}\right) - y^2\right)y' = y^2 - y\sec^2\left(\frac{x}{y}\right)$

$y' = \frac{y\sec^2\left(\frac{x}{y}\right) - y^2}{x\sec^2\left(\frac{x}{y}\right) + y^2} = \frac{y - y^2\cos^2\left(\frac{x}{y}\right)}{x + y^2\cos^2\left(\frac{x}{y}\right)}$

(3) [5 points] If $x^2 + y^2 + z^2 = 9$, $dx/dt = 5$ and $dy/dt = 4$ what is dz/dt when $(x, y, z) = (2, 2, 1)$? (This is Exercise 9 from Section 2.7. Imagine a particle moving along the surface of a sphere.)

Apply $\frac{d}{dt}$ to both sides.

$2xx' + 2yy' + 2zz' = 0$, Plug in $x=2, y=2, z=1, x'=5, y'=4$.

$2 \cdot 5 + 2 \cdot 4 + 1 \cdot z' = 0$

$z' = -18$

(4) [10 points] A spherical balloon is being filled with air at a steady rate of 4 cc/min (cc = cubic centimeters). What is the rate of change of its surface area when its radius is 3 cm? (This is from a Sample Final Exam: https://math.siu.edu/courses/test-preparation.php)

$V = \frac{4}{3}\pi r^3 \qquad S = 4\pi r^2$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4 \qquad \frac{dS}{dt} = 8\pi r \frac{dr}{dt} = ?$

$r=3$ ← given

$\frac{dr}{dt} = \frac{4}{4\pi r^2} = \frac{1}{9\pi}$

$\frac{dS}{dt} = 8\pi(3)\left(\frac{1}{9\pi}\right) = \frac{8}{3} = 2.66 \quad \frac{cm^2}{min}$