

Name: \_\_\_\_\_

Math 150

Quiz 6

Fall 2024

ONLY SCIENTIFIC CALCULATORS ALLOWED

(1) [5 points each] Compute the following.

a.  $(\sin(\ln x))'$

$$= \cos(\ln x) (\ln x)'$$

$$= \frac{\cos(\ln x)}{x}$$

b.  $(\ln \frac{1}{x})' = \frac{1}{\frac{1}{x}} (\frac{1}{x})'$

$$= x \left( \frac{-1}{x^2} \right) = -\frac{1}{x}$$

or,  $\ln \frac{1}{x} = \ln x^{-1} = -\ln x$   
 $(-\ln x)' = -\frac{1}{x}$

c.  $(e^{\tan x})'$

$$= e^{\tan x} (\tan x)'$$

$$= e^{\tan x} \sec^2(x)$$

d.  $(\cot^2(3x^2))' = \cancel{2 \cot(3x^2)} (\cancel{\csc(3x^2)})'$

$$= 2 \cot(3x^2) (-\csc^2(3x^2)) (3x^2)'$$

$$= -12x \cot(3x^2) \csc^2(3x^2) \quad \text{"6x"}$$

e.  $\lim_{x \rightarrow \infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

$$\lim_{x \rightarrow \infty} \frac{1 - e^{-6x}}{1 + e^{-6x}} = \frac{1 - 0}{1 + 0} = 1$$

This is #25 in 3.1

f.  $\lim_{x \rightarrow \infty} e^{-2x} \cos x$

This is #29 in 3.1

$$-1 \leq \cos x \leq 1$$

$$-e^{-2x} \leq e^{-2x} \cos x \leq e^{-2x}$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ 0 & & 0 \end{array}$$

Since  $-e^{-2x}$  and  $e^{-2x}$  go to 0, the squeeze theorem says  $e^{-2x} \cos x$  goes to 0 also.

(2) [10 points] This chemical reaction



has a reaction rate given by

$$\frac{d[N_2O_5]}{dt} = -0.0005[N_2O_5],$$

where  $[N_2O_5]$  is the concentration of  $N_2O_5$ . How long will it take for the concentration to be reduced to 90% of its original value? (This is Exercise 7 in Section 3.4.)

Let  $y(t) = [N_2O_5]$ .

$$\frac{dy}{dt} = -0.0005y$$

$$y = C e^{-0.0005t}$$

( $C = y(0)$ ), but  
we do not need this.

$$\text{Set } e^{-0.0005t} = 0.90$$

$$-0.0005t = \ln(0.90)$$

$$t = \frac{\ln(0.90)}{-0.0005} \approx 211 \text{ seconds} \\ (\text{time units})$$