Name: ______________________________

Math 150 Quiz 7 Fall 2024

ONLY SCIENTIFIC CALCULATORS ALLOWED

(1) [5 points each] Compute the follwing. Show work.

a. $(\sinh 3x + \cosh x^2)'$

$= \underset{\substack{\uparrow \\ (3x)'}}{3}\cosh(3x) + \underset{\substack{\uparrow \\ (x^2)'}}{\underline{2x}}\sinh(x^2)$

b. $\left(e^{\arcsin x}\right)'$

$= e^{\arcsin x} \cdot (\arcsin x)'$

$= \dfrac{e^{\arcsin x}}{\sqrt{1-x^2}}$

c. $(\tan^2 x + \tanh 3x)'$

$= 2\tan x(\tan x)' + \operatorname{sech}^2(3x)(3x)'$

$= 2\tan x \sec^2 x + 3\operatorname{sech}^2(3x)$

d. $(\sin(\cosh 3x))'$

$= \cos(\cosh(3x))(\cosh(3x))'$

$= \cos(\cosh(3x))\sinh(3x)(3x)'$

$= 3\cos(\cosh(3x))\sinh(3x)$

e. $\lim\limits_{x\to\infty} \ln \tanh x$

$= \ln\left(\lim\limits_{x\to\infty} \tanh x\right)$

$= \ln(1) = 0$

f. $\lim\limits_{x\to\infty} \dfrac{\cos 3x}{5x}$ $= 0$ since

$-1 \le \cos(3x) \le 1$

$-\dfrac{1}{5x} \le \dfrac{\cos(3x)}{5x} \le \dfrac{1}{5x}$

Since $\pm\dfrac{1}{5x} \to 0$ as $x \to \infty$, the Squeeze Thm says $\dfrac{\cos(3x)}{5x} \to 0$ as $x \to \infty$.

(2) [5 points] Find the equation of the tangent line to $y = \cosh(\sin x)$ at the point $(\pi, 1)$.

$y' = \sinh(\sin x)(\cos x)$.

Let $x = \pi$.

$y'(\pi) = \sinh(\sin \pi)(\cos \pi) = \sinh(0)\cdot(-1) = 0\cdot(-1) = 0$.

Thus, $y = 1$ is the tangent line.

(3) [5 points] Use implicit differentiation to find y' as a function of x and y for the relation $\tanh^2(xy^2) = 5$.

Apply $\frac{d}{dx}$ to both sides.

as long as not zero.

$$\cancel{2\tanh(xy^2)}\cdot \operatorname{sech}^2(xy^2)\,(xy^2)' = 0$$

$$y^2 + 2xyy' = 0$$

$$y' = \frac{-y}{2x}$$

But, this relation can never by satisfied! +2 bonus points if any one notices this.

(4) [5 points] Derive the formula for the derivative of $\arctan x$.

Let $\theta = \arctan x$.

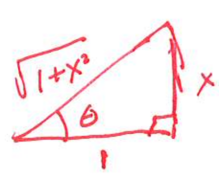
Thus, $\tan\theta = x$.

Apply $\frac{d}{dx}$.

$$\sec^2\theta \cdot \frac{d\theta}{dx} = 1$$

$$\frac{d\theta}{dx} = \cos^2\theta = \frac{1}{\sec^2\theta} = \frac{1}{1+\tan^2\theta} = \frac{1}{1+x^2}$$

Or, use triangle to show $\cos\theta = \frac{1}{\sqrt{1+x^2}}$.



$\cos\theta = \frac{1}{\sqrt{1+x^2}}$