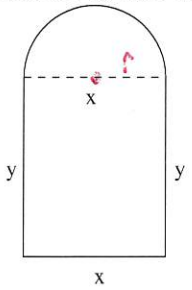
Name: _____

Math 150 Quiz 8 Fall 2024

ONLY SCIENTIFIC CALCULATORS ALLOWED

(1) [12 points] A Norman window has the shape of a rectangle surmonted by a semicircle. If the perimeter of the window is to be 30 ft, find the dimensions of the window so that the area enclosed is the maximum possible.



$P = 2y + x + \frac{1}{2}(2\pi(\frac{x}{2})) = 2y + (\frac{\pi}{2}+1)x$

Since $P=30$, $y = \frac{30-(\frac{\pi}{2}+1)x}{2} = 15 - (\frac{\pi}{4}+\frac{1}{2})x.$

$A = xy + \frac{1}{2}\pi(\frac{x}{2})^2 = xy + \frac{\pi}{8}x^2 = x\left(15-(\frac{\pi}{4}+\frac{1}{2})x\right) + \frac{\pi}{8}x^2$

$= 15x - (\frac{\pi}{4}+\frac{1}{2})x^2 + \frac{\pi}{8}x^2 = 15x - (\frac{\pi}{8}+\frac{1}{2})x^2.$

$\frac{dA}{dx} = 15 - 2(\frac{\pi}{8}+\frac{1}{2})x = 15 - (\frac{\pi}{4}+1)x.$ Thus, $\frac{dA}{dx} = 0$, when $x = \frac{15}{(\frac{\pi}{4}+1)}$.

or, $x = \frac{60}{\pi+4} \approx 8.4$ ft.

Next, $y = 15 - (\frac{\pi}{4}+\frac{1}{2})(\frac{60}{\pi+4}) = 15 - (\pi+2)(\frac{15}{\pi+4}) = 15\left(1 - \frac{\pi+2}{\pi+4}\right)$

$= 15\left(\frac{\pi+4}{\pi+4} - \frac{\pi+2}{\pi+4}\right) = 15\left(\frac{2}{\pi+4}\right) = \frac{30}{\pi+4} \approx$ 4.2 ft $(x=2y)$.

This is # 25 in Section 4.5.

This is #19 in Section 4.3.

(2) [10 points] Sketch the graph of a function that satisfies all of the given conditions.

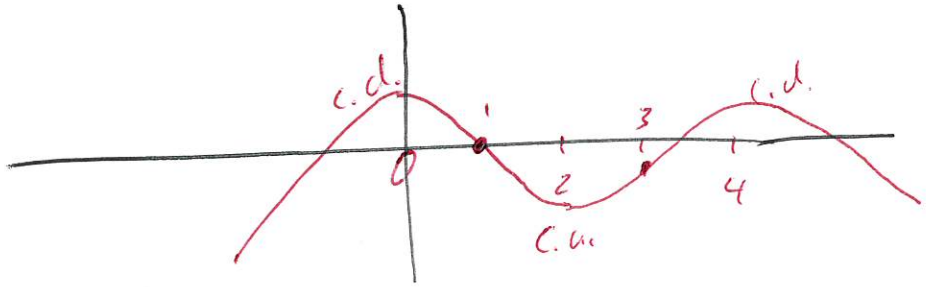
$f'(0) = f'(2) = f'(4) = 0$

$f'(x) > 0$ if $2 < x < 4$ or ~~$2 < x < 4$~~ $x < 0$.

$f'(x) < 0$ if $0 < x < 2$ or $x > 4$

$f''(x) > 0$ if $1 < x < 3$

$f''(x) < 0$ if $x < 1$ or $x > 3$



(3) [8 points] The velocity of an object is given by

$$v(t) = 3e^{2t} - \cosh(3t) + \tfrac{2}{t+1}.$$

a. Find the acceleration $a(t)$.

$a(t) = v'(t) = 6e^{2t} - 3\sinh(3t) - \frac{2}{(t+1)^2}$

b. Find the position, $s(t)$, assuming $s(0) = 3$.

$s(t) = \frac{3}{2}e^{2t} - \frac{1}{3}\sinh(3t) + 2\ln(t+1) + C$

$s(0) = \frac{3}{2} - 0 + 2\ln(1) + C = 3 \Rightarrow C = \frac{3}{2}$

(4) [10 points] Derive the formula for the derivative of $\arctan x$.

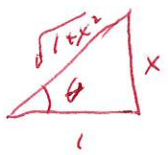
Let $\theta = \arctan x$.

Then $\tan\theta = x$.

Apply $\frac{d}{dx}$ to both sides.

$\sec^2\theta\,\theta' = 1 \quad (\theta' = \frac{d\theta}{dx})$

$\theta' = \cos^2\theta = \frac{1}{1+x^2}$



$$\frac{d\arctan x}{dx} = \frac{1}{1+x^2}.$$