Math 150 Quiz 9 Fall 2024

Name: ______________________________

1. [10 points] Find the general antiderivatives of the following. Don't forget the $+C$.

a. $3\sqrt{x} - 2\sqrt[3]{x}$

$= 3x^{\frac{1}{2}} - 2x^{\frac{1}{3}}$

antider. $= \dfrac{3X^{\frac{3}{2}}}{\frac{3}{2}} - \dfrac{2X^{\frac{4}{3}}}{\frac{4}{3}} + C$

$2X^{3/2} - \frac{3}{2}X^{\frac{4}{3}} + C$

b. $\int \sin t + 2\sinh t \, dt = -\cos t + 2\cosh t + C$

c. $\int \frac{4}{1+t^2}\, dt$

$= 4\arctan(x) + C$

d. $\int 2\cos t + \sec^2 t \, dt = 2\sin t + \tan t + C$

e. $\frac{x^5 - x^3 + 2x}{x^4}$

$= X - X^{-1} + 2X^{-3}$

antider. $= \frac{1}{2}X^2 - \ln|X| + \dfrac{2X^{-2}}{-2} + C$

$= \frac{1}{2}X^2 - \ln|X| - \dfrac{1}{X^2} + C$

f. $\int \sin 3x + \cos 2x \, dx = -\frac{1}{3}\cos(3x) + \frac{1}{2}\sin(2x) + C$

2. [10 points] Find a number c such that the function $f(x)$ defined below is always continuous.

$$f(x) = \begin{cases} \overbrace{x^2 - c}^{f_1(x)} & \text{if} \quad x < 3, \\ \underbrace{cx + 10}_{f_2(x)} & \text{if} \quad x \geq 3. \end{cases}$$

Need $f_1(3) = f_2(3)$.

$$9 - c = 3c + 10$$

$$-1 = 4c$$

$$c = -\frac{1}{4}$$

3. [10 points] Let $f(x) = \sqrt{2x+1}$. Use the definition of the derivative and the properties of limits to find $f'(x)$. Show all work.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}}$$

$$= \lim_{h \to 0} \frac{[\overbrace{2(x+h)}^{\cancel{2x}+2h} \cancel{+1}] - [\cancel{2x}\cancel{+1}]}{h\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)} = \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}\left(\sqrt{2(x+h)+1} + \sqrt{2x+1}\right)}$$

$$= \frac{2}{\sqrt{2(x+0)+1} + \sqrt{2x+1}} = \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$