Name: ______________________________

Math 150 Test 3 Fall 2024

ONLY SCIENTIFIC CALCULATORS ALLOWED

1. [7 points each] Do the following integrals. Don't forget the $+C$.

a. $\int \frac{\cos(\ln x)}{x}\, dx$ Let $u = \ln x$. Then $du = \frac{1}{x}\, dx$.

$= \int \cos(u)\, du = \sin(u) + C = \sin(\ln x) + C$

b. $\int \sec^2(e^x) e^x\, dx$ Let $u = e^x$. Then $du = e^x\, dx$

$= \int \sec^2(u)\, du = \tan(u) + C = \tan(e^x) + C$

c. $\int x^2 + \tan(3x)\, dx$ $= \frac{1}{3} x^3 + \int \tan(3x)\, dx$. Let $u = 3x$. Then $du = 3\,dx$

$= \frac{1}{3} x^3 + \frac{1}{3} \int \tan(3x) 3\, dx$

$= \frac{1}{3} x^3 + \frac{1}{3} \int \tan(u)\, du$

$= \frac{1}{3} x^3 + \frac{1}{3} \sec^2(u) + C$

$= \frac{1}{3} x^3 + \frac{1}{3} \sec^2(3x) + C$

2. [6 points] Set up the Riemann sum for $f(x) = \arctan x$ over $[0, 1]$, with 5 partition segments and right endpoints.

$\Delta x =$ $\frac{1-0}{5} = \frac{1}{5}$

$x_i =$ $0 + \frac{i}{5}$

Thus, the Riemann sum is ... $\sum_{i=1}^{5} \arctan\left(\frac{i}{5}\right) \cdot \frac{1}{5}$

3. [5 points] Suppose $g(x) = \int_0^{x^3} \sinh^5(t)\, dt$. Find $g'(x)$. Warning: Do not attempt to do the integral.

Suppose $F'(t) = \sinh^5(t)$.

Then $g'(x) = \left[F(x^3) - F(0)\right]' = F'(x^3)(3x^2) - 0$
$= 3x^2 \sinh^5(x^3)$

4. [5 points] The velocity of an object is given by $v(t) = 3t^2 + 2t$ in feet/second. How far will the object travel in 10 seconds?

$$\int_0^{10} 3t^2 + 2t\, dt = t^3 + t^2 \Big|_0^{10} = (1000 + 100) - (0 + 0)$$
$$= 1100.$$

Note: In this problem the velocity was never negative. Thus, the distance traveled and the displacement from the starting point were ~~are~~ the same. If $v(t)$ was sometimes negative then

distance traveled $= \int_0^{10} |v(t)|\, dt$, which you would need to break up, while

displacement $= \int_0^{10} v(t)\, dt$.

5. [6 points] Find two positive numbers such that $3x + 2y = 10$ and x^2y is a maximum.

$3x+2y=10 \Rightarrow y = \frac{10-3x}{2}$.

Let $f(x,y) = x^2y$. $f(\cancel{\;\;\;}) = f(x, \frac{10-3y}{2}) =$

$x^2\left(\frac{10-3x}{2}\right) = 5x^2 - \frac{3}{2}x^3$.

$\left(5x^2 - \frac{3}{2}x^3\right)' = 10x - \frac{9}{2}x^2 = \frac{1}{2}x(20-9x)$

This is zero for $x=0$ and $x = \frac{20}{9}$. Only $\frac{20}{9}$ is positive. x

Then $y = \frac{10 - 3\left(\frac{20}{9}\right)}{2} = 5 - \frac{10}{3} = \frac{15-10}{3} = \frac{5}{3}$. y

6. [13 points]

Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

(a) Evaluate $g(x)$ for $x = 0, 1, 2, 3, 4, 5$, and 6.

(b) Estimate $g(7)$.

(c) Where does g have a maximum value? Where does it have a minimum value?

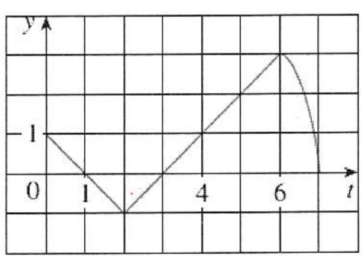
(d) Sketch a rough graph of g.

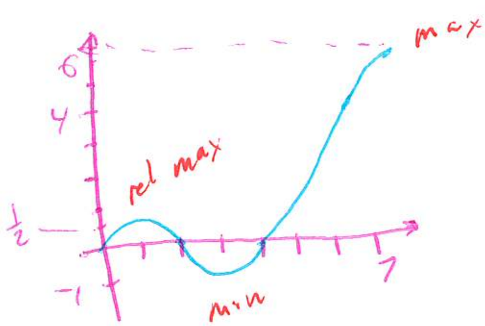
(a) $g(0)=0$
$g(1) = \frac{1}{2}$
$g(2) = 0$
$g(3) = -\frac{1}{2}$
$g(4) = 0$
$g(5) = 1\frac{1}{2}$
$g(6) = 4$

(b) $g(7) \approx 4 + 2\frac{1}{3} = 6\frac{1}{3}$
A little bigger than 6.

(c) max at $x=7$
min at $x=3$
rel max at $x=1$

(d)





7. [7 points each] Find the following definite integrals.

a. $\displaystyle\int_{-\pi/3}^{\pi/3} \frac{\tan x}{1+x^4}\,dx$ $\quad \dfrac{\tan(-x)}{1+(-x)^4} = \dfrac{-\tan(x)}{1+x^4} = -\dfrac{\tan(x)}{1+x^4}.$

Thus, the integrad is an odd function.
The interval is symmetric about 0.
Thus, this integral is 0.

b. $\displaystyle\int_0^1 x^3 - 3x^2 + e^x\,dx = \frac{x^4}{4} - 3\frac{x^3}{3} + e^x \Big|_0^1 = \left(\frac{1}{4} - 1 + e\right) - (0 - 0 + 1)$

$= -\frac{3}{4} + e - 1 = \frac{-7}{4} + e.$

c. $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx = \arctan(x)\Big|_0^1 = \arctan(1) - \arctan(0)$

$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$