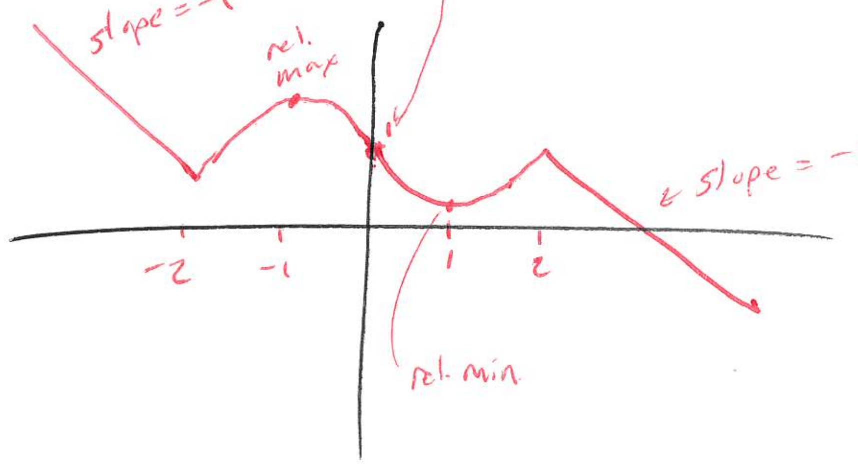
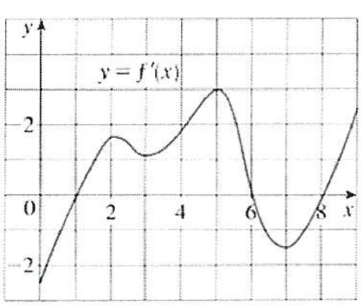
Name: ____________________

1. [10 points] Sketch the graph of a function that satifies all the given conditions.
$f'(1) = f'(-1) = 0$, $f'(x) < 0$ if $|x| < 1$, $f'(x) > 0$ if $1 < |x| < 2$,
$f'(x) = -1$ if $|x| > 2$
$f''(x) < 0$ if $-2 < x < 0$, and $(0, 1)$ is an inflection point.
(This is #20 in Section 4.3)



From the Fall 2023 Final Exam:

10. (12 points) Suppose that the DERIVATIVE f' of a function has the graph. (Assume the function f is defined only for $0 < x < \infty$.) Find the following:



a) Open interval where f is increasing: (1, 6) b/c $f'(x) > 0$ there.

b) State the x- coordinate of all extrema and label as max/min.

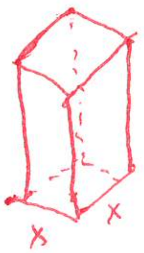
Max at $x =$ 6

Min at $x =$ 1, 8

c) Open interval(s) where f is concave up: (0, 2), (3, 5), (7, ∞)

d) State the x- coordinate of all Inflection points. $x =$ 2, 3, 5, 7

2. [10 points] Consider a box with a square base. If the surface area is 100 square ft, what dimensions maximize the volume?



$V = x^2 y$

$S.A. = 2x^2 + 4xy = 100$

Solve for y. $y = \frac{100 - 2x^2}{4x} = \frac{25}{x} - \frac{x}{2}$

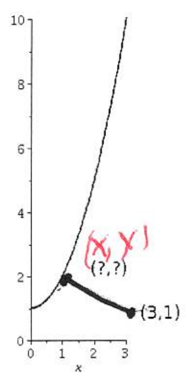
$V = x^2\left(\frac{25}{x} - \frac{x}{2}\right) = 25x - \frac{x^3}{2}.$

$\frac{dV}{dx} = 25 - \frac{3}{2}x^2 = 0$ when $x = \sqrt{\frac{50}{3}} \approx 4.0824$

Then $y = \frac{25}{\sqrt{\frac{50}{3}}} - \frac{\sqrt{\frac{50}{3}}}{2} = 4.0824\ldots$

Notice $x = y$.

3. [10 points] Find the point of the graph of $y = x^2 + 1$ that is closest to the point ~~(1, 3)~~. $(3,1)$



$d((x,y),(3,1)) = \sqrt{(x-3)^2 + (y-1)^2}$

But, it is easier to work with d^2.

$d^2 = (x-3)^2 + (y-1)^2$

$= x^2 - 6x + 9 + (x^2 + 1 - 1)^2$

$= x^4 + x^2 - 6x + 9.$

$(d^2)' = 4x^3 + 2x - 6.$

By inspection $x = 1$ gives $(d^2)' = 0$.

$(d^2)'' = 12x^2 + 2$. At $x = 1$, this is $14 > 0$, so $x = 1$ is a min of d^2.

If $x = 1$, then $y = 1^2 + 1 = 2$.

Thus $(1,2)$ is the pt on the graph closest to $(3,1)$