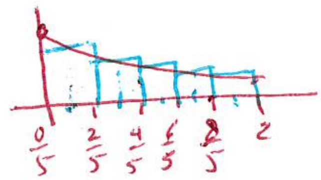
Name: ______________________

1. [10 points] Let A be the area of the region that lies under the graph of $f(x) = e^{-x}$ between $x = 0$ and $x = 2$. Estimate A using a Riemann sum with five subintervals using midpoints. Draw a picture representing this.

$\Delta x = \frac{2-0}{5} = \frac{2}{5} = 0.4.$



midpoints are

$x_1 = \frac{1}{5},\ x_2 = \frac{3}{5},\ x_3 = \frac{5}{5},\ x_4 = \frac{7}{5},\ x_5 = \frac{9}{5}$

0.2 0.6 1.0 1.4 1.8

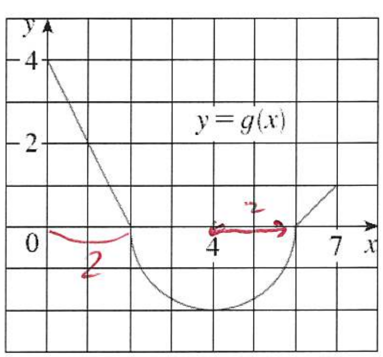
$$\sum_{i=1}^{5} e^{-x_i}\Delta x = \left(e^{-0.2} + e^{-0.6} + e^{-1.0} + e^{-1.4} + e^{-1.8}\right)\cdot(0.4) \approx 0.858927$$

Note: Exact value is $\int_0^2 e^{-x}\,dx = -e^{-x}\Big|_0^2 = (-e^{-2}) - (-e^{0}) = 1 - 1/e^2 \approx 0.8646647$

2. [8 points] The graph of $y = g(x)$ consists of two straight lines and a semicircle as shown below. Use it to evaluate each integral.

a. $\int_0^2 g(x)\,dx$ b. $\int_2^6 g(x)\,dx$

c. $\int_0^7 g(x)\,dx$ d. $\int_0^7 |g(x)|\,dx$



a. $\frac{2\cdot 4}{2} = 4$

b. $= -\frac{1}{2}\pi r^2 = -\frac{1}{2}\pi(2)^2 = -2\pi$

c. $4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$
≈ -1.783

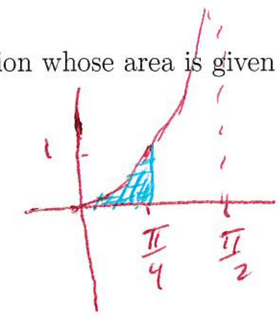
d. $4 + 2\pi + \frac{1}{2} \approx 10.783$

3. [12 points] a. Express the limit below as a definite integral.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \tan\left(i \cdot \frac{\pi}{4n}\right) \cdot \frac{\pi}{4n}$$

$$\int_0^{\frac{\pi}{4}} \tan x \, dx$$

b. Graph the region whose area is given by this integral.



c. Show that $(\ln(\sec x))' = \tan x$.

$$= \frac{1}{\sec x}(\sec x)' = \frac{\sec x \tan x}{\sec x} = \tan x \checkmark$$

d. Use the Evaluation Theorem to find the area of the region.

$$\int_0^{\pi/4} \tan x \, dx = \ln(\sec x)\Big|_0^{\pi/4} = \ln\left(\sec\frac{\pi}{4}\right) - \ln(\sec(0)) = \ln\sqrt{2} - \ln 1$$
$$= \ln\sqrt{2} \approx 0.3466$$

4. [10 points] From the Wayback Machine! Use the definition of the derivative to find:

$$\left(\frac{1}{2x+3}\right)' = \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{2(x+h)+3} - \frac{1}{2x+3}\right)$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{2x+3 - [2x+2h+3]}{(2(x+h)+3)(2x+3)}\right) = \lim_{h\to 0} \frac{1}{\cancel{h}} \frac{-2\cancel{h}}{(2(x+h)+3)(2x+3)}$$

$$= \frac{-2}{(2(x+0)+3)(2x+3)} = \frac{-2}{(2x+3)^2}$$