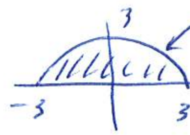
Math 150 Worksheet 13 Fall 2024

Name: ______________________________

1. [10 points] Find $\int_{-3}^{3} (x^7+1)\sqrt{9-x^2}\,dx$. Hint: Ask for help.

$$= \underbrace{\int_{-3}^{3} x^7\sqrt{9-x^2}\,dx}_{\text{odd}} + \int_{-3}^{3} \sqrt{9-x^2}\,dx \qquad x^2+y^2=9$$



$\dfrac{\pi 9}{2}$

$$= 0 + \frac{9\pi}{2} = \frac{9\pi}{2}$$

2. [5 points each]

a. $\int x\sqrt{4+x^2}\,dx$

$u = 4+x^2$

$du = 2x\,dx$

$$= \frac{1}{2}\int \sqrt{4+x^2}\; 2x\,dx$$

$$= \frac{1}{2}\int u^{\frac{1}{2}}\,du = \frac{1}{2}\,\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{1}{3}(4+x^2)^{3/2} + C$$

b. $\int \frac{e^x}{1+e^x}\,dx$

$u = 1+e^x$

$du = e^x\,dx$

$$= \int \frac{1}{u}\,du = \ln|u| + C = \ln(1+e^x) + C$$

c. $\int \frac{\sin x}{3-\cos x}\,dx$

$u = 3-\cos x$

$du = \sin x\,dx$

$$\int \frac{1}{u}\,du = \ln(3-\cos x) + C$$

d. $\int \frac{2x}{1+x^4}\,dx$

$u = 1+x^4$ won't work. Would get $du = 4x^3\,dx$. Yikes!

let $u = x^2$. $du = 2x\,dx$.

$$\int \frac{1}{1+u^2}\,du = \arctan(u) + C$$

$$= \arctan(x^2) + C$$

3. [10 points]

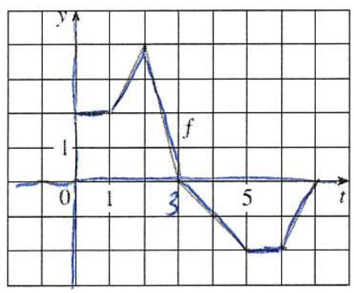
Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph is shown.

(a) Evaluate $g(0)$, $g(1)$, $g(2)$, $g(3)$, and $g(6)$.

(b) On what interval is g increasing?

(c) Where does g have a maximum value?

(d) Sketch a rough graph of g.



(a) $g(0)=0$

$g(1)=2$

$g(2)=2+3=5$

$g(3)=5+2=7$

$g(4)=7-\frac{1}{2}=6.5$

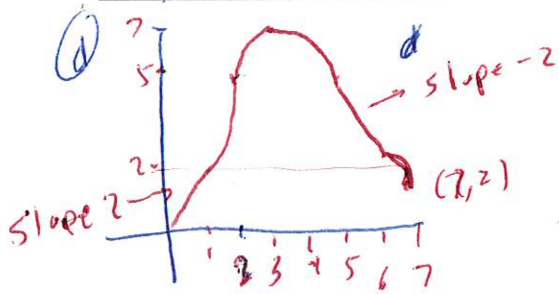
$g(5)=6.5-1.5=5$

$g(6)=5-2=3$

$g(7)=3-1=2$

	inc	dec.
(b)	$(0,3)$	$(3,7)$

(c) $x=3$



4. [5 points] Let $g(x) = \int_{\ln x}^{x^2} \text{myst}(t)\,dt$. Here myst is the *mystery* function. Find $g'(x)$.

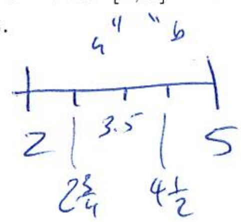
$= F(x^2) - F(\ln x)$, where $F'(x) = \text{myst}(x)$.

Then $g'(x) = [F(x^2) - F(\ln x)]'$

$= 2x\,F'(x) - \dfrac{F'(\ln x)}{x}$

$= 2x\,\text{myst}(x^2) - \dfrac{\text{myst}(\ln x)}{x}$

5. [5 points] Set up the Riemann sum for $f(x) = x^3$ over $[2, 5]$ with 4 subintervals using midpoints. Do not evaluate it.



$\Delta x = \frac{5-2}{4} = \frac{3}{4}$

$x_i = 2 + \frac{\Delta x}{2} + i\Delta x$

$$\sum_{i=0}^{3} \left(2 + \frac{3}{8} + i\frac{3}{4}\right)^3 \left(\frac{3}{4}\right).$$

6. [5 points each] Do the following limits.

a. $\lim\limits_{x\to\infty} 2x - \sqrt{4x^2 - x}$

$$\frac{2x - \sqrt{4x^2-x}}{1} \cdot \frac{2x + \sqrt{4x^2-x}}{2x + \sqrt{4x^2-x}} = \lim \frac{4x^2 - (4x^2 - x)}{2x + \sqrt{4x^2-x}}$$

$$= \lim_{x\to\infty} \frac{x}{2x + \sqrt{4x^2-x}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{1}{2 + \sqrt{4 - \frac{1}{x}}} = \frac{1}{2+\sqrt{4-0}} = \frac{1}{4}$$

b. $\lim\limits_{t\to 0} \dfrac{\cos 2t}{\sec 3t}$ $= \dfrac{\cos(2\cdot 0)}{\sec(3\cdot 0)} = \dfrac{\cos(0)}{\sec(0)} = \dfrac{1}{1} = 1.$