Math 150 Worksheet 2 Fall 2024

Name:

1. [2 points] Write down the formal definition of $\lim_{x\to c} f(x) = L$. (Assume $L$ and $c$ are finite.)

For every $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x-c| < \delta$, then $|f(x) - L| < \varepsilon$.

2. [10 points] Use the formal definition to show that $\lim_{x\to 2} 3x + 7 = 13$.

Let $\varepsilon > 0$. Let $\delta = \varepsilon/3$. Suppose $0 < |x-2| < \frac{\varepsilon}{3}$.

Then $-\frac{\varepsilon}{3} < x - 2 < \frac{\varepsilon}{3}$.

Thus, $-\varepsilon < 3x - 6 < \varepsilon$

Thus $13 - \varepsilon < 3x + 7 < \varepsilon + 13$

Thus $-\varepsilon < f(x) - 13 < \varepsilon$.

Thus $|f(x) - 13| < \varepsilon$.

Thus $\lim_{x\to 2} 3x+7 = 13$.

3. [10 points] Use the formal definition to show that $\lim_{x\to 1} -2x + 5 = 3$.

Let $\varepsilon > 0$. Let $\delta = \varepsilon/2$. Suppose $0 < |x-1| < \delta$.

Then $-\frac{\varepsilon}{2} < x - 1 < \frac{\varepsilon}{2}$.

Thus, $\varepsilon > -2x + 2 > -\varepsilon$

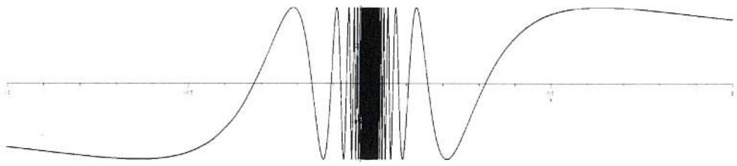
Thus, $\varepsilon + 3 > -2x + 5 > 3 - \varepsilon$.

Thus $\varepsilon > f(x) - 3 > -\varepsilon$

Thus, $|f(x) - 3| < \varepsilon$.

Thus, $\lim_{x\to 1} f(x) = 3$.

4. [20 points] Let $f(x) = \sin \frac{1}{x}$. We will show that $\lim_{x \to 0} f(x)$ does not exit. This will be done using *proof by contradiction.* Suppose the limit does exist and is equal to $L$.



a. Suppose $L \geq 0$. Let $\epsilon = 1$. Let $\delta$ be any positive number. Find a value for $x \neq 0$ between $-\delta$ and $\delta$ such that $f(x) = -1$. Consider $|f(x) - L|$. What does this tell you?

$\sin\left(\frac{3\pi}{2} + 2\pi n\right) = -1$ for all $n \in \mathbb{Z}$.

Let $x_n = \frac{1}{\frac{3\pi}{2} + 2\pi n}$. Then $\sin(x_n) = -1$.

Choose $n$ big enough that $\frac{1}{\frac{3\pi}{2} + 2\pi n} < \delta$. Clearly $x_n > 0$.

Now for this $x_n$

$|\sin(x_n) - L| = |-1-L| = 1+L \geq 1 = \varepsilon$. So $|f(x_n) - L|$ is not less than $\varepsilon$, even though $0 < |x_n - 0| < \delta$.

b. Suppose $L < 0$. Let $\epsilon = 1$. Let $\delta$ be any positive number. Find a value for $x \neq 0$ between $-\delta$ and $\delta$ such that $f(x) = 1$. Consider $|f(x) - L|$. What does this tell you?

Let $x_n = \frac{1}{\frac{\pi}{2} + 2\pi n}$. Then $f(x_n) = 1$. Choose $n$ big enough that $x_n < \delta$. Clearly $x_n > 0$. But

$|f(x_n) - L| = |1 - L| > 1$ since $L < 0$,

Thus $|f(x_n) - L| > \varepsilon$ and not $< \varepsilon$ even though $0 < |x_n - 0| < \delta$.

c. Could our supposition that the limit existed possibly be true?

No way!