Name: ______________________

Math 150 — Worksheet 4 — Fall 2024

1. [20 points] Find the following limits. Show your work.

a. $\displaystyle\lim_{x\to\infty}\sqrt{x^4+x^2}-x^2$

"$\infty-\infty$"

$$\frac{\sqrt{x^4+x^2}-x^2}{1}\cdot\frac{\sqrt{x^4+x^2}+x^2}{\sqrt{x^4+x^2}+x^2}$$

$$=\lim_{x\to\infty}\frac{x^4+x^2-x^4}{\sqrt{x^4+x^2}+x^2}=\lim_{x\to\infty}\frac{x^2}{\sqrt{x^4+x^2}+x^2}\cdot\frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x^2}}+1}=\frac{1}{\sqrt{1+0}+1}=\frac{1}{2}$$

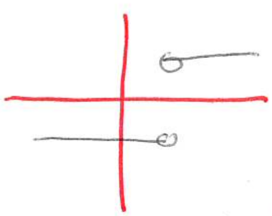
b. $\displaystyle\lim_{x\to 0}\frac{\cos 3x\tan 2x}{\sin 5x}$

"$\frac{0}{0}$"

$$=\lim_{x\to 0}\cos 3x\,\frac{5x}{5\sin 5x}\,\frac{2\tan 2x}{2x}$$

$$=\frac{2}{5}\lim_{x\to 0}\cos 3x\cdot\frac{5x}{\sin 5x}\,\frac{\sin 2x}{2x}\,\frac{1}{\cos 2x}$$

$$=\frac{2}{5}\cdot 1\cdot 1\cdot 1\cdot\frac{1}{1}=\frac{2}{5}$$



c. $\displaystyle\lim_{x\to 1}\frac{|x-1|}{x-1}$

Does not exist.

Note: $\displaystyle\lim_{x\to 1^+}\frac{|x-1|}{x-1}=1$

$\displaystyle\lim_{x\to 1^-}\frac{|x-1|}{x-1}=-1$

These do not match so limit as $x\to 1$ does not exist.

d. $\displaystyle\lim_{x\to 1}\frac{x^3-1}{x-1}$

$\frac{0}{0}$

$$\begin{array}{r|l} & x^2+x+1 \\ x-1 & x^3-1 \\ & -(x^3-x^2) \\ & x^2-1 \\ & -(x^2-x) \\ & x-1 \end{array}$$

$$=\lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{x-1}$$

$$=\lim_{x\to 1}x^2+x+1=1+1+1=3$$

2. [20 points] Find the following limits. Show your work.

$\frac{\infty}{\infty}$

a. $\lim_{x\to\infty} \frac{7x^3+x+1}{3x^3+x^2+9}$ $\frac{\frac{1}{x^3}}{\frac{1}{x^3}}$

$= \lim_{x\to\infty} \frac{7+\frac{1}{x^2}+\frac{1}{x^3}}{3+\frac{1}{x}+\frac{9}{x^3}}$

$= \frac{7+0+0}{3+0+0} = \frac{7}{3}$

b. $\lim_{x\to 2} \frac{7x^2+2}{3x+1}$ $= \frac{7\cdot 4+2}{6+1} = \frac{30}{7}$

c. $\lim_{x\to 0} 3x\csc 5x$

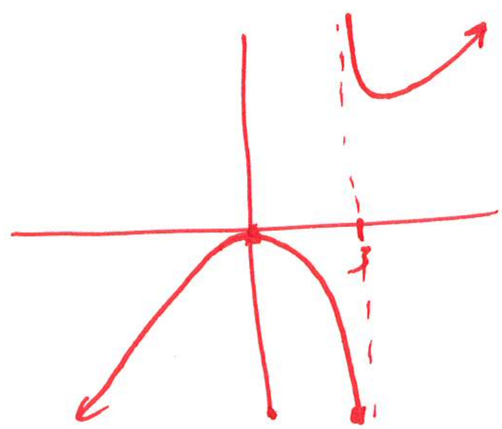
$= \lim_{x\to 0} \frac{3x}{\sin 5x} = \lim_{x\to 0} \frac{3}{5}\,\frac{5x}{\sin 5x}$

$= \frac{3}{5}\cdot 1 = \frac{3}{5}$

d. $\lim_{x\to 3^-} \frac{x^2}{x-3}$

"$\frac{9}{0}$" = "$\pm\infty$"

For $x<3$, $x-3$ is negative. The top is never neg. So limit is $-\infty$



It will have an oblique asymptote

3. [15 points] From a height on 200 ft you throw a ball downward at 10 ft/sec. When will it hit the ground? How fast will it be going? (Use $a = -32$ ft/sec/sec.)

$a = -32$

$v = -32t + v_0 \qquad v_0 = -10$

$s = -16t^2 - 10t + s_0 \qquad s_0 = 200$

$s(t) = -16t^2 - 10t + 200 = 0$

$$t = \frac{10 \pm \sqrt{100 - 4(-16)200}}{-32} = \frac{10 \pm \sqrt{6500}}{-32}$$

$$= \frac{10 \pm 80.6}{-32} = -2.83 \text{ seconds or } +2.21 \text{ seconds}$$

disgard

$v(2.21) = -80.6$ ft/sec or 80.6 ft/sec downward.

4. [5 points] Find the equation of the line tangent to $y = 2x^2 + x - 1$ at the point $(2, 9)$.

$y' = 4x + 1 \qquad y'(2) = 9.$

t. line is $y - 9 = 9(x - 2)$

$y = 9x - 18 + 9$

$y = 9x - 9.$

5. [20 points] Find the derivatives from definition using only the rules for limits.

a. $f(x) = \frac{1}{\sqrt{x+1}}$

$$f' = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h+1}} - \frac{1}{\sqrt{x+1}}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x+1}-\sqrt{x+h+1}}{h\left(\sqrt{x+h+1}\sqrt{x+1}\right)} \cdot \frac{\sqrt{x+1}+\sqrt{x+h+1}}{\sqrt{x+1}+\sqrt{x+h+1}}$$

$$= \lim_{h\to 0} \frac{x+1-(x+h+1)}{h\left(\sqrt{x+h+1}\sqrt{x+1}\right)\left(\sqrt{x+1}+\sqrt{x+h+1}\right)}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}(\quad)(\quad)} = \lim_{h\to 0} \frac{-1}{\sqrt{x+h+1}\sqrt{x+1}\left(\sqrt{x+1}+\sqrt{x+h+1}\right)}$$

$$= \frac{-1}{\sqrt{x+1}\sqrt{x+1}\left(\sqrt{x+1}+\sqrt{x+1}\right)}$$

$$= \frac{-1}{2(x+1)\sqrt{x+1}}$$

b. $f(x) = x^3$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 = 3x^2 + 0 + 0 = 3x^2$$

6. [20 points] Find the derivatives.

a. $f(x) = \sin\pi + x^2 \tan 3x$.

$\cancel{f'(x) = 0 + 2x\tan 3x} +$

$f'(x) = (\sin\pi)' + (x^2\tan(3x))'$

constant.

$(\sin\pi = 0)$

$= 0 + (x^2)'\tan(3x) + x^2(\tan 3x)'$

$= 2x\tan(3x) + x^2\sec^2(3x)\cdot(3x)'$

$= 2x\tan(3x) + 3x^2\sec^2(3x)$

b. $f(x) = \frac{3x+2}{5x+7}$. $\cancel{f' = (3x+2)'(5x+7)\ (3x+2)(}$

$f' = \dfrac{(3x+2)'(5x+7) - (3x+2)(5x+7)'}{(5x+7)^2}$

$= \dfrac{3(5x+7) - (3x+2)\cdot 5}{(\ \ )^2}$

$= \dfrac{15x + 21 - 15x - 10}{(\ \ )^2}$

$= \dfrac{11}{(5x+7)^2}$

Notice $f'(x)$ is never zero.

c. $f(x) = 7^2 + x^{-3} + \sin x^2$.

constant

$f'(x) = 0 + (x^{-3})' + (\sin(x^2))'$

$= -3x^{-4} + \cos(x^2)\cdot(x^2)'$

$= -3x^{-4} + 2x\cos(x^2)$

d. $g(x) = \sec x + \sqrt{7x+2}$

$g'(x) = \sec x\tan x + \dfrac{7}{2\sqrt{7x+2}}$

$(\sqrt{7x+2})' = \dfrac{1}{2\sqrt{7x+2}}(7x+2)'$

$= \dfrac{7}{2\sqrt{7x+2}}$