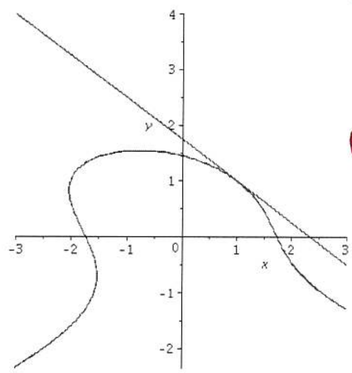
Math 150 Worksheet 5 Fall 2024

Name:

1. [5 points] Consider the equation $x^2 + xy + y^3 = 3$. Find the equation of the line tangent to its graph at the point (1,1). Express your answer in slopt-intercept form.



Apply $\frac{d}{dx}$ to both sides of $x^2+xy+y^3=3$.

$$2x+y+xy'+3y^2y'=0$$

Let $x=1, y=1$.

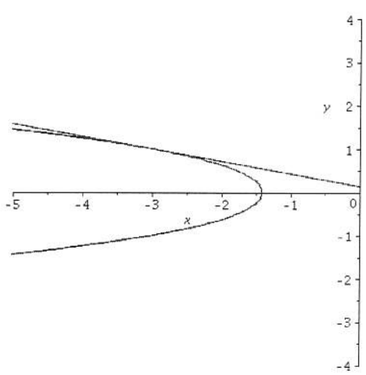
$$3+y'+3y'=0$$

$$y'=-\frac{3}{4}$$

Find tangent line: $(y-1)=-\frac{3}{4}(x-1)$

or $y=-\frac{3}{4}x+\frac{7}{4}$

2. [5 points] Consider the equation $x^2 + 2xy^2 - y^2 = 2$. Find the equation of the line tangent to its graph at the point $(-3, 1)$. Express your answer in slopt-intercept form.



Apply $\frac{d}{dx}$ ...

$$2x+2y^2+4xyy'-2yy'=0$$

Let $x=-3, y=1$.

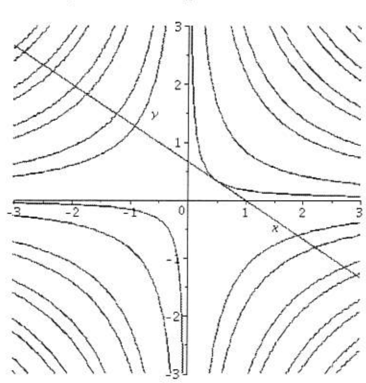
$$-6+2-12y'-2y'=0$$

$$y'=\frac{4}{-14}=-\frac{2}{7}$$

Tangent line: $y-1=-\frac{2}{7}(x+3)$

$$y=-\frac{2}{7}x+\frac{1}{7}$$

3. [5 points] Consider the equation $\sin(\pi xy) = \frac{1}{2}$. Find the equation of the line tangent to its graph at the point $(\frac{1}{2}, \frac{1}{3})$. Express your answer in slopt-intercept form.



Apply $\frac{d}{dx}$ ...

$$\cos(\pi xy)(\pi y+\pi xy')=0$$

$x=\frac{1}{2}$, $y=\frac{1}{3}$: $\cancel{\pi\cos(\frac{\pi}{6})}(\frac{1}{3}+\frac{1}{2}y')=0$

$$y'=-\frac{2}{3}$$

T-line $y-\frac{1}{3}=-\frac{2}{3}(x-\frac{1}{2})$

$$y=-\frac{2}{3}x+\frac{2}{3}$$

4. [10 points] Air is leaking out of a spherical ballon at a rate of 200 cc / min. When the radius of the ballon is 100 cm, what is the rate of change of the radius?

$$V = \frac{4}{3}\pi r^3$$

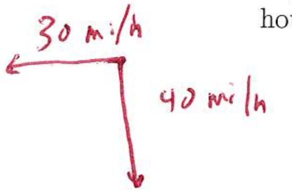
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \qquad \frac{dV}{dt} = -200 \text{ (given)}$$

$$r = 100 \text{ (given)}$$

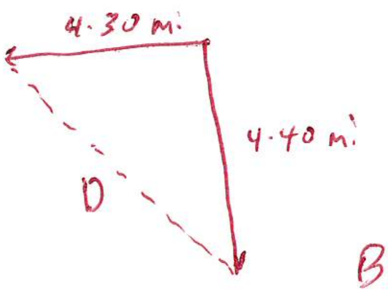
Thus $-200 = 4\pi (100)^2 \frac{dr}{dt}$.

Hence, $\frac{dr}{dt} = \frac{-200}{4\pi(10{,}000)} \approx -0.00159$ cm/min

5. [10 points] Two cars start moving from the same point at the same time. One travels south at 40 mi/h and the other travels west at 30 mi/h. At what rate is the distance between the cars increasing four hours later? Bonus: Does the answer actually depend on the time?



After 4 hours



I'll use x for west distance traveled and y for south dist. traveled.

$$D^2 = x^2 + y^2 \qquad 2DD' = 2xx' + 2yy' \quad \left(\text{prime} = \frac{d}{dt}\right)$$

$$D' = \frac{xx' + yy'}{D} = \frac{xx' + yy'}{\sqrt{x^2+y^2}}$$

$$= \frac{120 \cdot 30 + 160 \cdot 40}{\sqrt{(120)^2 + (160)^2}} = \frac{10{,}000}{200} = 50 \text{ mi/h.}$$

Bonus: No!

$$D'(t) = \frac{(tx')x' + (ty')y'}{\sqrt{(tx')^2 + (ty')^2}} = \frac{(x')^2 + (y')^2}{\sqrt{(x')^2 + (y')^2}}$$

$$= \sqrt{(x')^2 + (y')^2} = \sqrt{(30)^2 + (40)^2} = 50 \text{ mi/h}$$

For more examples see: https://tutorial.math.lamar.edu/classes/calci/relatedrates.aspx (Related Rates, Pauls Online Math Notes)

6. [5 points each] Find the following.

a. $(x^3 \sin 2x)'$

$(x^3)' \sin(2x) + x^3(\sin 2x)'$

$3x^2 \sin(2x) + 2x^3 \cos(2x)$

b. $\lim_{x\to -3} \frac{x^3+9x^2+27x+27}{x^2+6x+9}$ $\frac{0}{0}$

$$\begin{array}{r} x+3 \\ x^2+6x+9 \enclose{longdiv}{x^3+9x^2+27x+27} \\ -(x^3+6x^2+9x) \\ \hline 3x^2+18x+27 \\ 3x^2+18x+27 \\ \hline 0 \end{array}$$

Thus $x^3+9x^2+27x+27=(x+3)(x^2+6x+9)$

$\lim_{x\to -3}$ ———— $= \lim_{x\to -3} x+3 = 0$

c. $(\cot^2 x^4)'$

$2\cot(x^4)\cdot(-\csc(x^4)\cot(x^4))\cdot 4x^3$

$= -8x^3\csc(x^4)\cot^2(x^4)$

d. $\left(\frac{x^2+x}{2x+3}\right)'$

$= \frac{(x^2+x)'(2x+3)-(x^2+x)(2x+3)'}{(2x+3)^2}$

$= \frac{(2x+1)(2x+3)-2(x^2+x)}{(2x+3)^2}$

$= \frac{4x^2+7x+4-2x^2-2x}{(2x+3)^2}$

$= \frac{2x^2-5x+4}{(2x+3)^2}$

e. $\frac{d^{64}}{dx^{64}} \sin x$

Since $64 = 4\cdot 16$

The answer is

$\sin x$

f. $\frac{d^{82}}{dx^{82}} \cos x$

Since $82 = 4\cdot 20+2$

$\frac{d^{82}}{dx^{82}}\cos x = \frac{d^2 \cos x}{dx^2} = -\cos x$