Math 150 Worksheet 6 Fall 2024

Name: _____

1. [2 points each] Compute the following.

a. $\log_7 49$ $= \log_7 7^2 = 2$

b. $\log 1,000,000$ $= \log_{10} 10^6 = 6$

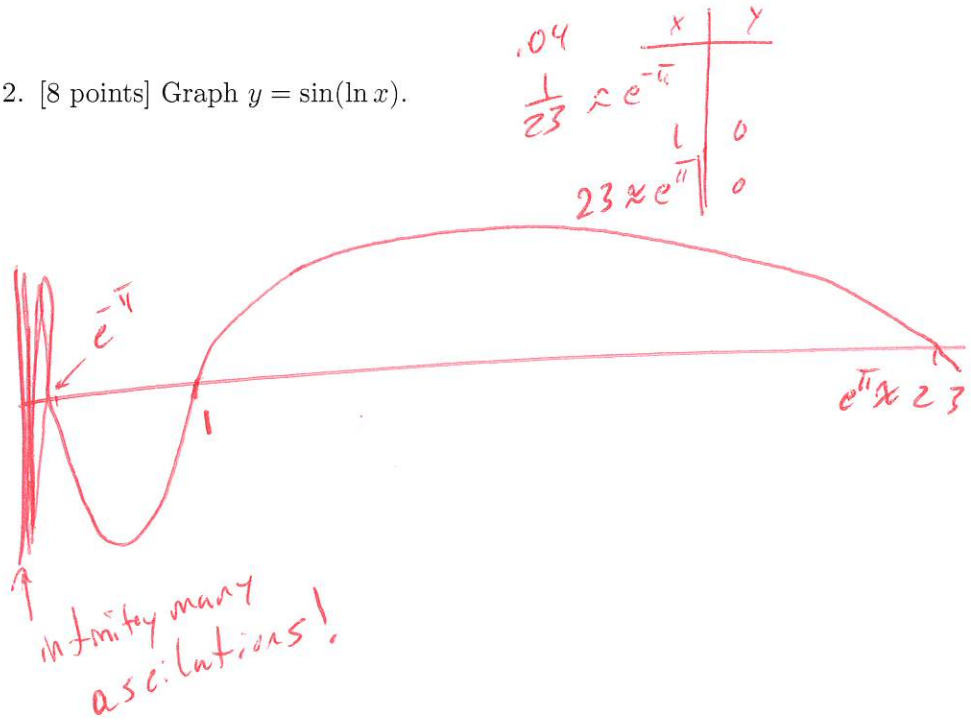
c. $\log_2 0.25$ $= \log_2 (2)^{-2} = -2$

d. $e^{3 \ln 2}$ $= e^{\ln 2^3} = 2^3 = 8$

e. $\ln(e^4)$ $= 4$

f. $10^{\log \log_3 9}$ $= \log_3 9 = \log_3 3^2 = 2$

2. [8 points] Graph $y = \sin(\ln x)$.



3. [10 points] An ancient wooden tool found at an archaeological site was found to contain 1.67% of the original amount of carbon-14. (This is done by measuring the ratio of carbon-12 to carbon-14.) The half-life of carbon-14 is 5730 years. How old was the wooden tool? (Round to the nearest century.)

The model is $C(t) = C_0 e^{-rt}$.

At $t = 5730$, $e^{-r(5730)} = \frac{1}{2}$. $r = \frac{-\ln(\frac{1}{2})}{5730}$

Given $e^{-rt} = .0167$

$t = \frac{\ln(.0167)}{-r} \approx 33800$ years ago.

4. [10 points] An object is heated to 100° C and is then allowed to cool in a room whose air temperature is 30° C. If the temperature is 80° C after 5 minutes, when will its temperature be 50° C?

The model is $T(t) = T_a + (T_0 - T_a) e^{-kt}$.

$= 30 + (100 - 30) e^{-kt}$

$= 30 + 70 e^{-kt}$.

Find k. $80 = 30 + 70 e^{-k5}$

$e^{-k5} = \frac{50}{70} = \frac{5}{7}$

$k = \frac{-\ln 5/7}{5} \approx -0.673$

Find time.

$50 = 30 + 70 e^{-kt}$

$e^{-kt} = \frac{20}{70} = \frac{2}{7}$

$t = \frac{-\ln \frac{2}{7}}{k} \approx 18.6$ minutes

5. [5 points each] Compute the following derivatives.

a. $\left(e^{\tan x^2}\right)'$

$e^{\tan x^2} \cdot (\tan x^2)'$

$= e^{\tan x^2} \sec^2 x^2 \cdot (x^2)'$

$= 2x \sec^2(x^2)\, e^{\tan(x^2)}$

b. $(x^2 \ln \cot x)'$ $= 2x \ln(\cot(x)) + \frac{x^2}{\cot x}(-\csc^2 x)$

$= 2x \ln[\cot(x)] - x^2 \csc^2 x \tan x$

c. $(xe^{x^3})''$

$= (e^{x^3} + xe^{x^3}(3x^2))'$

$= [(1+3x^3)e^{x^3}]'$

$= 9x^2 e^{x^3} + (1+3x^3)(3x^2)e^{x^3}$

$= (9x^5 + 12x^2)e^{x^3}$

d. $(x^{\cos x})'$

let $y = x^{\cos x}$

$\ln y = \ln x^{\cos x} = \cos x \ln x$

$\frac{y'}{y} = -\sin x \ln x + \frac{\cos x}{x}$

$y' = x^{\cos x}\left(\frac{\cos x}{x} - \sin x \ln x\right)$

e. $(\ln e^{\sin x})'$

$= (\sin x)'$

$= \cos x$

$\left(\frac{x+\ln x}{e^x \sin x}\right)' = \frac{\left(1+\frac{1}{x}\right)e^x \sin x - (x+\ln x)(e^x \sin x + e^x \cos x)}{(e^x \sin x)^2}$