Math 150 — Worksheet 7 — Fall 2024

Name: ______________________

1. [5 points each] Compute the following. Show work.

a. $((\tanh 2x)^x)'$

Let $y = (\tanh(2x))^x$

Then $\ln y = \ln[(\tanh(2x))^x]$

$= x \ln(\tanh(2x))$

$\frac{y'}{y} = \ln(\tanh(2x)) + \frac{x}{\tanh(2x)} \cdot \operatorname{sech}^2(2x) \cdot 2$

$y' = (\tanh(2x))^x \cdot (\ )$

b. $(\arcsin(e^{2x}))'$

$\frac{1}{\sqrt{1-(e^{2x})^2}} (e^{2x})'$

$= \frac{2e^{2x}}{\sqrt{1-e^{4x}}}$

c. $(\cosh x \arctan x)'$

$= (\cosh x)'(\arctan x) + (\cosh x)(\arctan x)'$

$= \sinh x \arctan x + \frac{\cosh x}{1+x^2}$

d. $\lim_{x\to\infty} 3x - \sqrt{9x^2-3x+1}$

$\frac{3x - \sqrt{9x^2-3x+1}}{1} \cdot \frac{3x + \sqrt{\ }}{3x+\sqrt{\ }}$

$= \lim_{x\to\infty} \frac{\cancel{9x^2} - (\cancel{9x^2} - 3x + 1)}{3x+\sqrt{\ }} = \lim \frac{3x-1}{3x+\sqrt{\ }} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{3 - \frac{1}{x}}{3 + \sqrt{9 - \frac{3}{x} + \frac{1}{x^2}}} = \frac{3-0}{3+\sqrt{9}}$

$= \frac{3}{6} = \frac{1}{2}$

e. $\lim_{x\to-\infty} \sin(\arctan 3x)$

$= \sin\left(\lim_{x\to-\infty} \arctan(3x)\right)$

$= \sin\left(-\frac{\pi}{2}\right) = -1$

f. $\log_3(27)^4$

$= 4\log_3 27 = 4\log_3 3^3 = 4 \cdot 3 = 12$

g. $\lim_{x\to 0} \arccos\left(\frac{\sin x}{2x}\right)$

$= \arccos\left(\lim_{x\to 0} \frac{\sin x}{2x}\right)$

$= \arccos\left(\frac{1}{2} \lim_{x\to 0} \frac{\sin x}{x}\right)$

$= \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$

h. $(x^2 \arccos 3x)' = (x^2)' \arccos(3x) + x^2(\arccos(3x))'$

$= 2x\arccos(3x) - \frac{x^2 \cdot 3}{\sqrt{1-9x^2}}$

2. [10 points] The velocity of an object is given by

$$v(t) = 3t^2 + \frac{1}{t^2+1} + 4e^{2t} + \sinh t.$$

a. Find its acceleration, $a(t)$. $= v'(t)$.

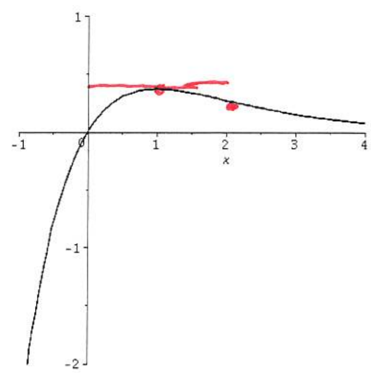
$$a(t) = 6t - \frac{2t}{(t^2+1)^2} + 8e^{2t} + \cosh(t)$$

b. Find its position, $s(t)$, assuming $s(0) = 0$. Find $s(t)$ such that $s'(t) = v(t)$.

$$s(t) = t^3 + \arctan(t) + 2e^{2t} + \cosh(t) + s_0.$$

$$s(0) = 0 + 0 + 2 + 1 + s_0 = 0. \text{ Thus } s_0 = -3.$$

3. [10 points] Let $f(x) = xe^{-x}$. See graph below.




a. For what value of $x$ is the tangent line horizontal?

$f'(x) = e^{-x} - xe^{-x} = (1-x)e^{-x}$. This is zero when $x=1$.

b. For what value of $x$ is $f''(x) = 0$?

$f''(x) = (-1)e^{-x} - (1-x)e^{-x} = (x-2)e^{-x}$. This is zero when $x=2$.