Math 150 Worksheet 8 Fall 2024

Name: ______________________

1. [5 points each] Compute the following. Show work.

a. $\lim_{x\to 0} \frac{\sec(3x)}{\sec(5x)}$ $= \frac{\sec(0)}{\sec(0)} = \frac{1}{1} = 1$

b. $\lim_{x\to 0} \frac{\tan(3x)}{\tan(5x)}$ Indeterminate Form $\frac{0}{0}$ Use L'Hôp.

$= \lim_{x\to 0} \frac{(\tan(3x))'}{(\tan(5x))'} = \lim_{x\to 0} \frac{3\sec^2(3x)}{5\sec^2(5x)} = \frac{3\cdot\sec(0)}{5\sec(0)}$

$= \frac{3\cdot 1}{5\cdot 1} = \frac{3}{5}$

Has form 1^∞

c. $\lim_{x\to 0} (1+5x)^{1/x}$ $= L$

$\ln L = \lim_{x\to 0} \ln(1+5x)^{1/x} = \lim_{x\to 0} \frac{\ln(1+5x)}{x}$ $\frac{0}{0}$

Apply L'Hôp.

$\lim_{x\to 0} \frac{\frac{1}{1+5x}\cdot 5}{1} = \frac{5}{1+5\cdot 0} = 5.$

Thus $\ln L = 5$, so $L = e^5$

d. $\lim_{x\to 0} \frac{1}{x} - \frac{1}{\sin x}$ Form is $\infty - \infty$.

$= \lim_{x\to 0} \frac{\sin x - x}{x\sin x}$ $\frac{0}{0}$ Apply L'Hôp.

$= \lim_{x\to 0} \frac{\cos x - 1}{\sin x + x\cos x}$ still $\frac{0}{0}$. Apply L'Hôp again.

$= \lim_{x\to 0} \frac{-\sin x}{\cos x + \cos x - x\sin x} = \frac{-0}{1+1-0} = \frac{0}{2} = 0$

e. $\lim_{x\to\infty} x^x$

Form is ∞^∞.

This is not indeterminate!

Once $x > 1$, $x^x > x$

so limit is ∞.

f. $\lim_{x\to\infty} \frac{e^{2x}+x^2+2}{3e^{2x}+7x^2+x}$ Form is $\frac{\infty}{\infty}$.

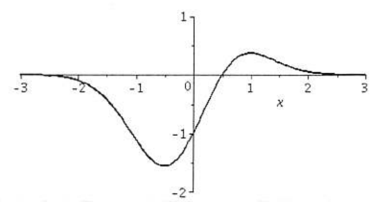
Apply L'Hôp.

$\lim_{x\to\infty} \frac{2e^{2x}+2x}{6e^{2x}+14x+1}$ still $\frac{\infty}{\infty}$. Apply L'Hôp again.

$\lim_{x\to\infty} \frac{4e^{2x}+2}{12e^{2x}+14}$ still $\frac{\infty}{\infty}$. Again!

$\lim_{x\to\infty} \frac{8e^{2x}}{24e^{2x}} = \lim_{x\to\infty} \frac{8}{24} = \frac{1}{3}$

2. [10 points] Let $f(x) = (2x-1)e^{-x^2}$. See graph below.



a. [4 points] Find $f'(x)$. $= 2e^{-x^2} - 2x(2x-1)e^{-x^2}$

$$= (-4x^2+2x+2)e^{-x^2}$$

b. [2 points] For what values of x is $f'(x) = 0$? Since e^{-x^2} is never zero, divid through by it to get

$$-4x^2+2x+2 = 0$$

or $2x^2 - x - 1 = 0$

$$x = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} = 1, -\frac{1}{2}$$

c. [2 points] What is the absolute minimum value of $f(x)$? Give the exact value and the numerical rounded value to 3 decimal places.

Min is at $x = -\frac{1}{2}$

$f(-\frac{1}{2}) = -2e^{-1/4} \approx -1.558$

d. [2 points] What is the absolute maximum value of $f(x)$? Give the exact value and the numerical rounded value to 3 decimal places.

Max is at $x = 1$.

$f(1) = e^{-1} \approx 0.368$