Example similar to #54 in Section 4.7

The driver of a stopped car hits the gas and accelerates at 10 m/s^2 for 5 seconds. He then cruises for 1 minute at a constant speed. Then he hits the breaks and slows the car down linearly until it stops after 10 seconds. How far did the car go? Plot $a(t)$, $v(t)$ and $s(t)$ for $0 \le t \le 75$ seconds.

Solution. We divide the trip into three phases.

Phase I. $0 \le t \le 5$

$$\begin{aligned} a_1(t) &= 10 \\ v_1(t) &= 10t + v_0 \\ s_1(t) &= 5t^2 + v_0 t + s_0 \end{aligned}$$

Clearly, $v_0 = 0$ and $s_0 = 0$.

Phase II. $5 \le t \le 65$

$$\begin{aligned} a_2(t) &= 0 \\ v_2(t) &= C \\ s_2(t) &= Ct + D \end{aligned}$$

How to find C and D? At $t = 5$ we have $v_2(5) = v_1(5) = 50$. Thus, $C = 50$. To find D we use the continuity requirement that $s_2(5) = s_1(5)$. Thus,

$$50 \cdot 5 + D = 5(5)^2.$$

Solving for D we get $D = -125$. We now have the following equations for Phase II.

$$\begin{aligned} a_2(t) &= 0 \\ v_2(t) &= 50 \\ s_2(t) &= 50t - 125 \end{aligned}$$

Phase III. $65 \le t \le 75$

We know is that the velocity is linear. Let $v_3(t) = At + B$. Now we can write

$$\begin{aligned} a_3(t) &= A \\ v_3(t) &= At + B \\ s_3(t) &= (A/2)t^2 + Bt + C \end{aligned}$$

We know that $v_3(65) = v_2(65) = 50$ and $v_3(75) = 0$. Since we have two data points we can determine the line. The slope is $A = (0-50)/(75-65) = -5$. Next $v_3(75) = -5 \cdot 75 + B = 0$ so $B = 375$.

Now we work on $s_3(t)$. We know $s_3(65) = s_2(65)$. Thus

$$(-5/2)(65)^2 + 375 \cdot 65 + C = 50 \cdot 65 - 125,$$

which gives

$$C = 3,125 + 10,562.5 - 24,375 = -10,687.5.$$

We therefore have

$$\begin{aligned} a_3(t) &= -5 \\ v_3(t) &= -5t + 375 \\ s_3(t) &= (-5/2)t^2 + 375t - 10,687.5 \end{aligned}$$

We can now compute that $s_3(75) = 3375$ meters.

Finally we collect our results and plot them.

$$a(t) = \begin{cases} 10 & \text{for} \quad 0 \le t \le 5 \\ 0 & \text{for} \quad 5 \le t \le 65 \\ -5 & \text{for} \quad 65 \le t \le 75 \end{cases}$$

$$v(t) = \begin{cases} 10t & \text{for} \quad 0 \le t \le 5 \\ 50 & \text{for} \quad 5 \le t \le 65 \\ -5t + 375 & \text{for} \quad 65 \le t \le 75 \end{cases}$$

$$s(t) = \begin{cases} 5t^2 & \text{for} \quad 0 \le t \le 5 \\ 50t - 125 & \text{for} \quad 5 \le t \le 65 \\ (-5/2)t^2 + 375t - 10,687.5 & \text{for} \quad 65 \le t \le 75 \end{cases}$$

□

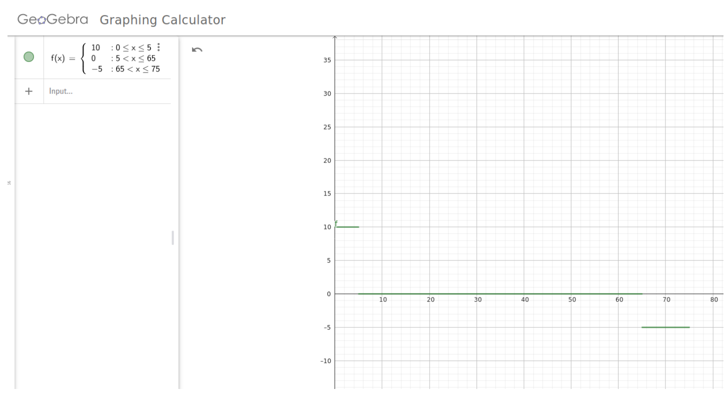

FIGURE 1. Acceleration

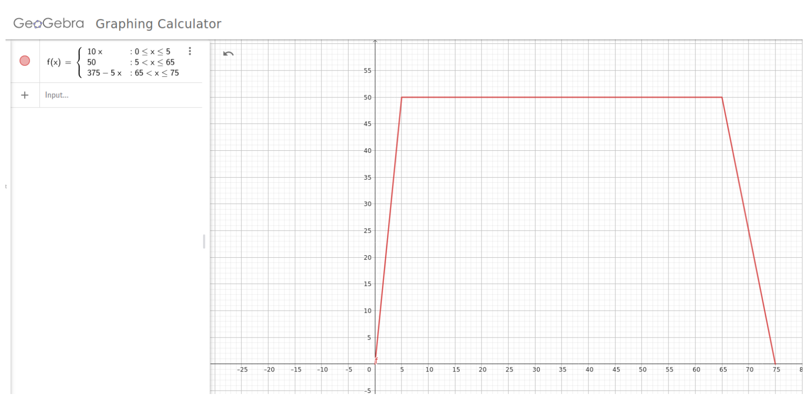

FIGURE 2. Velocity

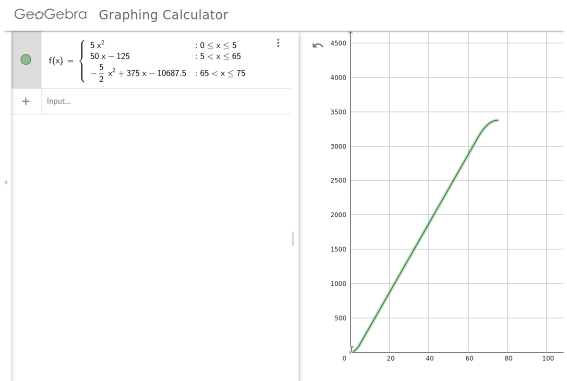

FIGURE 3. Position: notice how the concavity changes