ONLY SCIENTIFIC CALCULATORS ALLOWED

Test 1 is Friday, September 20. It covers Sections 1.3 - 2.5. It will be 5 pages, 20 points per page.

Limits are hard because there is not one simple algorithm you can memorize and apply. You have to stop and THINK. What is the function doing near the limit value? For example,

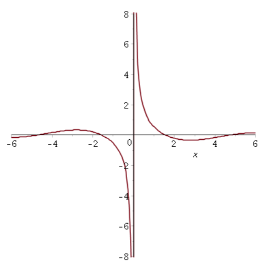
$$\lim_{x\to 0} \frac{\sec(5x^2)}{\sec(3\tan(7x))}$$

looks really complicated, but it is not! Remember, $\sec(0) = 1/\cos(0) = 1/1 = 1$. Since $\tan(0) = 0$, this limit is just $1/1 = 1$.

However, this limit

$$\lim_{x\to 0^+} \frac{\cos x}{x}$$

looks simple, but is rather subtle. $\cos(0) = 1$, but $1/x \to \infty$ as $x \to 0$ from the right (the positive side). So, this limit is ∞. Here is the graph.



Notice the limit as $x \to 0^-$ (from the negative side) is $-\infty$, while the limit as $x \to \infty$ is zero.

When a function behaves badly near the limiting value, that is if it of one of the **indeterminate forms**, $0/0$, ∞/∞, $0 \cdot \infty$, or $\infty - \infty$, then some algebraic trickery may be called for. This requires lots of practice.

Example. Find $\lim_{x\to 0} \frac{1-\cos x}{x}$. Notice it tends to $0/0$. The trick here is

$$\lim_{x\to 0} \frac{1-\cos x}{x} \cdot \frac{1+\cos x}{1+\cos x} = \lim_{x\to 0} \frac{1-\cos^2 x}{x(1+\cos x)}$$
$$= \lim_{x\to 0} \frac{\sin^2 x}{x(1+\cos x)} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1+\cos x}$$
$$= \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{\sin x}{1+\cos x} = 1 \cdot \frac{0}{1+1} = 0.$$

Example. Find $\lim_{x\to\infty} \sqrt{x^2+x} - x$. Notice it tends to $\infty - \infty$. The trick here is

$$\lim_{x\to\infty} \frac{\sqrt{x^2+x}-x}{1} \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x} = \lim_{x\to\infty} \frac{(x^2+x)-(x^2)}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{x^2+x}+x} \cdot \frac{1/x}{1/x} = \lim_{x\to\infty} \frac{1}{\sqrt{1+\frac{1}{x}}+1} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}.$$

Problems where you use a graph to find limits were on Quiz 2. See also Exercises 3, 4, and 6 in Section 1.3, 1 and 2, in 1.6 and 19 in the Chapter 1 Review.

Continuity. In Section 1.5 be able to find where a given function is continuous. Be able to find parameters that make a function continuous, like exercises 33 and 34 (33 was on a quiz). Be able to use the Intermediate Value Theorem (see Example 8, that was on a quiz, and Exercises 39-42).

Derivatives. Be able to find the derivative using the definition as on Quiz 4. Be able to find equation of tangent lines.

Example. Find tangent line for $f(x) = 5x^2 + x^3$ at $(1, 6)$.

Solution. $f'(x) = 10x + 3x^2$. Thus, $f'(1) = 10 + 3 = 13$. Therefore, the tangent line is

$$y - 6 = 13(x - 1)$$

or, in slope-intercept form

$$y = 13x - 7.$$

Be able to graph $f'(x)$ given the graph of $f(x)$, like on Worksheet 3.

Be able to do motion problems like on Quiz 4.

Know how to compute derivatives using the Product, Quotient and Chain Rules.

Know the derivatives of the six trig functions **or** be able to derive them quickly using the Quotient Rule. See table at bottom of page 111 in textbook.

Good luck! Study hard. See me if you have questions!