Example. Find $\lim_{\theta \to 0} \dfrac{\tan 2\theta \sin 3\theta}{\theta^2}$.

Solution.

$$\begin{aligned}
\lim_{\theta \to 0} \frac{\tan 2\theta \sin 3\theta}{\theta^2} &= \lim_{\theta \to 0} \frac{2 \sin 2\theta}{2\theta} \cdot \frac{1}{\cos 2\theta} \cdot \frac{3 \sin 3\theta}{3\theta} \\
&= 6 \left(\lim_{\theta \to 0} \frac{\sin 2\theta}{2\theta} \right) \left(\lim_{\theta \to 0} \frac{1}{\cos 2\theta} \right) \left(\lim_{\theta \to 0} \frac{\sin 3\theta}{3\theta} \right) \\
&= 6 \cdot 1 \cdot \tfrac{1}{1} \cdot 1 = 6.
\end{aligned}$$

□

Question. What is this good for?

Suppose $f(\theta) = \tan 2\theta \sin 3\theta$ was a model for some process and that the application only required small values of θ. Well, for small values of θ we now know that

$$f(\theta) \approx 6\theta^2.$$

This is a lot easier to work with. If you were writing computer code where such a calculation would be repeated thousands or millions of times, replacing $f(\theta)$ with $6\theta^2$ would save computational time. If you were designing a chip to implement this you might not have tan and sin functions like you would if coding in C++ or Python. Also, simplified coding not only runs faster, but generates less waste heat from the chip. You have probably read that these big data centers use up a lot of electricity. Much of this is for the cooling systems needed to prevent chips from overheating. So, more efficient coding not only saves time, but is good for the environment!

On the next page are graphs of $f(\theta)$ and $6\theta^2$, overlaid, so you can compare them.

Graph with GeoGebra.

Graph with Maple.