

Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \text{ where } x_0 \text{ in an initial guess.}$$

Integral Formulas

$$1. \int a^u du = \frac{a^u}{\ln a} + C$$

$$2. \int \sec u du = \ln |\sec u + \tan u| + C$$

$$4. \int \sec^2 u du = \tan u + C$$

$$6. \int \sec u \tan u du = \sec u + C$$

$$3. \int \csc u du = -\ln |\csc u + \cot u| + C$$

$$5. \int \csc^2 u du = -\cot u + C$$

$$7. \int \csc u \cot u du = -\csc u + C$$

Numerical Integration

1. Trapezoidal Rule:

$$\int_b^a f(x) dx \approx T(n) \equiv \frac{b-a}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$$

Also,  $T(n) = \frac{1}{2}[L(n) + R(n)]$ , where  $L(n)$  and  $R(n)$  are the left end point and right end point Riemannian sums, respectively.

2. Simpson's Rule:

$$\int_b^a f(x) dx \approx S(n) \equiv \frac{b-a}{3n} [f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

Also,  $S(n) = \frac{1}{3}[T(n/2) + 2M(n/2)]$ , where  $M(n/2)$  is the midpoint Riemannian sum and  $T(n/2)$  is defined above. Note that  $n$  must be even in Simpson's Rule.

Volumes of Revolution

$$1. \text{ Disk Method: } V = \pi \int_b^a [R(x)]^2 dx$$

$$2. \text{ Washer Method: } V = \pi \int_b^a ([R(x)]^2 - [r(x)]^2) dx$$

3. Shell Method:  $V = 2\pi \int_b^a p(x)h(x) dx$ , where  $p(x)$  is the radius of the shells and  $h(x)$  is the height of the shells.

Arc Length

$$L = \int_b^a \sqrt{1 + [f'(x)]^2} dx$$

Surface Area of Revolution

$$S = 2\pi \int_b^a r(x)\sqrt{1 + [f'(x)]^2} dx$$

Note: The formulas for volume, length, and surface area all have analogs for  $dy$  integration.