

NO CALCULATORS

Name: _____ ID #: _____

Part I: Calculations

1. [20 points] Find the inverse of $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 0 \end{bmatrix}$. Show all steps.

2. [20 points] Find all values of x such that $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & x \\ 1 & 3 & 2 \end{bmatrix}$ is singular.

3. [20 points] Find an equation for the plane in R^3 that contains the point $(1,1,1)$ and the y -axis.

4. [20 points] Find the solution of the initial value problem $v'(t) = Av(t)$ where

$$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}, \quad v = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} \quad \text{and} \quad v(0) = \begin{bmatrix} 4 \\ 4 \end{bmatrix}.$$

5. [20 points] Apply the Gram Schmidt process to the set $\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ 0 \end{bmatrix} \right\}$.

6. [20 points] Find the LU decomposition of $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & -1 \end{bmatrix}$.

7. [20 points] Find the coordinates of $\begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$ with respect to the basis $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$.

8. [20 points] Find the equation of the quadratic function that passes through the three points, $(-1, 1)$, $(1, 2)$, and $(2, 4)$.

Part II: Theory

9. [10 points] Define the following terms:

a) Linear independence.

b) Rank of a matrix.

c) Basis of vector space.

d) Eigenvalue and eigenvector.

e) Null space of a matrix.

10. [10 points] Let A be an $m \times n$ matrix with rank n . Prove that $A^T A$ is nonsingular.

11. [10 points] Let A and B be 2×2 matrices. Is it possible for $AB - BA$ to be equal to the identity matrix? Prove your claim.

12. [10 points] Let A be a nonsingular matrix. Suppose that λ is an eigenvalue of A . Prove that $1/\lambda$ is an eigenvalue of A^{-1} .