

Print Name: _____ Id number: _____

NO CALCULATORS

1. [20 points] Let $V = \text{span}\{e^x, xe^x, x^2e^x\}$. Let $D : V \rightarrow V$ be differentiation, i.e. $D(f) = f'$, for all $f \in V$. Find the matrix representing D with respect to the basis above for V .

2. [20 points] Find the inverse of $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 0 \end{bmatrix}$. Show all steps. Hint: check your answer.

3. [20 points] Find the coordinates of $\begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} \in \mathbb{R}^3$ with respect to the basis

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}.$$

Be sure to check your answer.

4. [20 points] Apply the Gram-Schmidt process to the set below. Hint: check that your result is an orthonormal set.

$$\left\{ \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 6 \\ 0 \end{bmatrix} \right\}$$

5. [20 points] Find the equation of the quadratic function that passes through $(-1, 1)$, $(1, 2)$ and $(2, 4)$.
6. [20 points] Find a basis for the solution space of the system of linear differential equations

$$x'(t) = x(t) + 2y(t)$$

$$y'(t) = x(t) + y(t)$$

Find the particular solution when $x(0) = 1$ and $y(0) = 2$.

7. [20 points] Which sets below are vector spaces? Briefly justify your answers.

a) All continuous functions from \mathbb{R} to \mathbb{R} .

b) All functions $f : \mathbb{R} \rightarrow \mathbb{R}^2$ of the form $f(x) = (ax, x + b)$.

c) All 3×3 symmetric matrices.

d) The solution set to $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

e) The solution set to $2x - 3 = 0$.

8. [20 points] Which functions/transformations below are linear? Briefly justify your answers.

a) $f(x, y) = x^2 + y^2$.

b) $f(x, y, z) = 3x - y + 3^2z$.

c) $f(x) = \ln(2^x)$.

d) $L(f) = f'' + f'$.

e) $L(f) = 2f' + 1$.

9. [20 points] Prove that if A is an upper triangular matrix, then the eigenvalues of A are the elements on the main diagonal of A .

10. [20 points] Prove that if A is a nonsingular square matrix, then

$$(A^T)^{-1} = (A^{-1})^T.$$